VELOCITY AND ACCELERATION ESTIMATION IN VIDEO SEQUENCES BY THE LOCAL POLYNOMIAL PERIODOGRAM

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ABSTRACT

Motion parameters estimation is performed by using the local polynomial periodogram. The presented method provides the estimation of velocity and acceleration, simultaneously. It is appropriate for the motion parameters estimation in noisy sequences. Theoretical considerations are illustrated by numerical examples.

1. INTRODUCTION

Motion parameter estimation is an important topic in digital video data processing [1, 2]. Spectral analysis based method is an approach used for this purpose [3, 4, 5]. This method is based on the SLIDE algorithm. In the case of constant velocity the Fourier transform can be used, while for the estimation of time-varying velocity, time-frequency distributions are required [6, 7]. Different time-frequency distributions can be applied, depending on the specific case. Although the SLIDE algorithm based on the constant μ propagation function is the simplest for this purpose, the SLIDE algorithm with variable μ propagation has been used in order to estimate two parameters of motion (initial position and velocity) . In this paper we will use the local polynomial periodogram [8] and SLIDE algorithm with constant μ propagation, which will provide the simultaneous estimation of velocity and acceleration of a moving object. The local polynomial periodogram is linear, with respect to a signal, and is appropriate for signal analysis in noisy environments.

2. THEORETICAL BACKGROUND

In order to obtain 1D signals containing information about motion parameters, which will be also appropriate for the time-frequency analysis, the frame projection to the coordinate axes and the SLIDE algorithm is used [3, 9]. Consider a video sequence in the form:

$$i(x,y,t) = f(x,y) + s(x-\varphi_x(t), y-\varphi_y(t)), \qquad (1)$$

where $f(x,y)$ is the background, and $s(x-\varphi_x(t), y-\varphi_y(t))$ is a moving object. Obviously the functions $\varphi_x(t)$ and $\varphi_y(t)$ describe the position of the object in the $x-$ and $y-$coordinate, respectively. Projections of the frames are:

$$P_x(x,t) = \sum_y i(x,y,t) \qquad P_y(y,t) = \sum_x i(x,y,t). \qquad (2)$$

By using the SLIDE algorithm we obtain:

$$z_x(t) = \sum_x (\partial P(x,t)/\partial x) e^{j\mu x}. \qquad (3)$$

After relatively simple manipulations we obtain (see [3, 6, 9]):

$$z_x(t) = \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k \varphi_x(t)}{dt^k} F_x(\mu) e^{j\mu\varphi_x(t)} = A(t) e^{j\mu\varphi_x(t)}. \qquad (4)$$

Parameter $A(t)$ is the amplitude of the signal $z_x(t)$ which depends of the motion parameters, while $F_x(\mu)$ is the Fourier transform of $\partial f(x-\varphi_x(t))/\partial x$.

The ideal time-frequency representation of the signal $z_x(t)$ would produce:

$$ITF_x(t,\omega) = 2\pi A(t)^2 \delta(\omega - \mu d\varphi_x(t)/dt). \qquad (5)$$

Thus, the instantaneous frequency gives the velocity of the object, i.e., $v_x(t) = d\varphi_x(t)/dt = \omega/\mu$. Note that the previous results hold for the $x-$component of the velocity parameter. Equation for the $y-$component follow in a straightforward manner:

$$ITF_y(t,\omega) = 2\pi A(t)^2 \delta(\omega - \mu d\varphi_y(t)/dt). \qquad (6)$$

It is well known that the choice of a specific time-frequency distribution is crucial in motion parameter estimation. The Wigner distribution based estimation is more accurate than the spectrogram based one [6, 7, 10]. In this way we estimate time-varying velocity, but if additional information is required (for example the initial position of the object) some modifications in the SLIDE algorithm are necessary. In this paper we use the local polynomial periodogram which provides the estimation of the time-varying velocity and time-varying acceleration without any modification in the structure of the SLIDE algorithm.

3. LOCAL POLYNOMIAL PERIODOGRAM

The local polynomial Fourier transform is introduced by Katkovnik [8], and defined as:

$$LP(t,\vec{\omega}) = \int_{-\infty}^{\infty} z_x(t+\tau)w(\tau)e^{-j\theta(\vec{\omega},\tau)}d\tau \qquad (7)$$

where the function $\theta(\vec{\omega},\tau)$ is:

$$\theta(\vec{\omega},\tau) = \omega_1\tau + \omega_2\tau^2/2! + \omega_3\tau^3/3! + ... + \omega_n\tau^n/n!. \quad (8)$$

From the above definition we can conclude that the local polynomial transform is a $(n+1)$-dimensional transform and concentrated along $\vec{\omega} = (d\varphi(t)/dt, d^2\varphi(t)/dt^2, ..., d^n\varphi(t)/dt^n)$ where $d\varphi(t)/dt$ is the instantaneous frequency, and $d^2\varphi(t)/dt^2$, ..., $d^n\varphi(t)/dt^n$ are its first and higher order derivatives. In our application the second order local polynomial transform with $\theta(\vec{\omega},\tau) = \omega_1\tau + \omega_2\tau^2/2$ is used:

$$LP(t,\omega_1,\omega_2) = \int_{-\infty}^{\infty} z_x(t+\tau)w(\tau)e^{-j\omega_1\tau - j\omega_2\tau^2/2}d\tau. \qquad (9)$$

Note that the local polynomial Fourier transform of the second order can be easily connected to the fractional Fourier transform, [11]. From (9), for $\omega_2 = 0$, the short-time Fourier transform follows:

$$STFT(t,\omega) = \int_{-\infty}^{\infty} z_x(t+\tau)w(\tau)e^{-j\omega\tau}d\tau. \qquad (10)$$

The spectrogram is the squared modulo of the short-time Fourier transform, i.e., $SPEC(t,\omega) = |STFT(t,\omega)|^2$. The Wigner distribution can be defined as [10]:

$$WD(t,\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} STFT(t,\omega+\theta)STFT^*(t,\omega-\theta)d\theta. \qquad (11)$$

The local polynomial periodogram is defined by:

$$LPP(t,\omega_1,\omega_2) = |LP(t,\omega_1,\omega_2)|^2. \qquad (12)$$

Thus, the time-frequency representation, of the signal $z_x(t)$, by using the local polynomial periodogram will be:

$$\begin{aligned} LP_x(t,\omega_1,\omega_2) &= 2\pi A(t)^2\delta(\omega_1 - \mu d\varphi_x(t)/dt) \\ \text{for} \quad \omega_2 &= \mu d^2\varphi_x(t)/dt^2. \end{aligned} \qquad (13)$$

It is assumed that $d^k\varphi_x(t)/dt^k = 0$ for $k > 2$. The local polynomial periodogram is highly concentrated along the instantaneous frequency (instantaneous frequency is proportional to the object velocity) for ω_2 given by (11) which is proportional to the object acceleration. The local polynomial periodogram is a linear and very appropriate tool for the time-frequency analysis of noisy signals, as well. This property has practical importance in the case of variable structures of backgrounds in video sequences. The adaptive form of the local polynomial Fourier transform can be written as:

$$LP(t,\omega,\hat{a}) = \int_{-\infty}^{\infty} z_x(t+\tau)w(\tau)e^{-j\hat{a}\tau^2}e^{-j\omega\tau}d\tau \quad (14)$$

where $\hat{a}$ is the parameter from a considered set that produces the highest concentration:

$$\hat{a} = \arg\max_{a\in\mathbf{A}}\{M(t,a)\}. \qquad (15)$$

As measure of the concentration we will use:

$$M(t,a) = \frac{\int_{-\infty}^{\infty} |LP(t,\omega,a)|^2 d\omega}{\left|\int_{-\infty}^{\infty} LP(t,\omega,a)d\omega\right|^{3/2}}. \qquad (16)$$

4. NUMERICAL EXAMPLES

Example 1: Consider an artificial video sequence consisting of a moving object 12×12 pixels size and static background with Gaussian noise ($N\times N$, $N = 256$). Number of frames is 100. The initial position of the object is: $(m,n) = (120,2)$ while the object motion can be described by: $\varphi_x(t) = m + 0.02t^2$. The positions of the object in frames 1, 50 and 100 are shown in the Fig 1. The spectrogram (squared modulo of the short-time Fourier transform), local polynomial periodogram (of the second order) and the Wigner distribution are calculated by using the window $w(\tau)$ with $N = 64$. Zero padding with $N_z = N$ and $\mu = 0.6$ are used. The time-frequency representation of signal $z_x(t)$, using the spectrogram, Wigner distribution and the local polynomial periodogram are depicted in Fig.2. We can see from Fig.3 that the velocity estimation by using the Wigner distribution and local polynomial periodogram are almost the same. They are of significantly better accuracy than velocity estimation based on the spectrogram. In addition, the local polynomial distribution provides an acceleration estimation, Fig.4.

Example 2: Here, we use the same parameters as in Example 1, only the background is variable. Namely, temporally and spatially white Gaussian noise as background with variance $\sigma^2 = 0.0144$ is generated. In this case we have a high amount of noise. Time-frequency representations are given in Fig.5. In Fig.6 the velocity

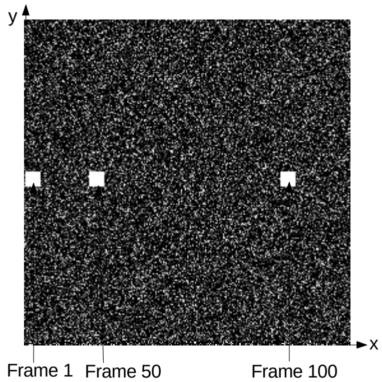

Figure 1: Positions of the object in Frames 1, 50 and 100.

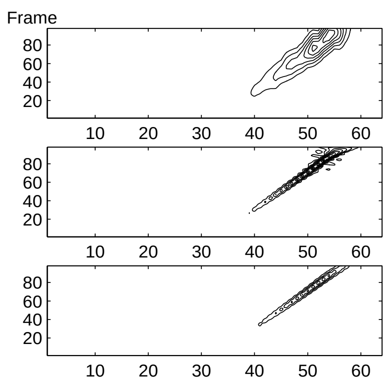

Figure 2: Time-frequency representation based on: a) Spectrogram; b) Wigner distribution; c) Local polynomial periodogram.

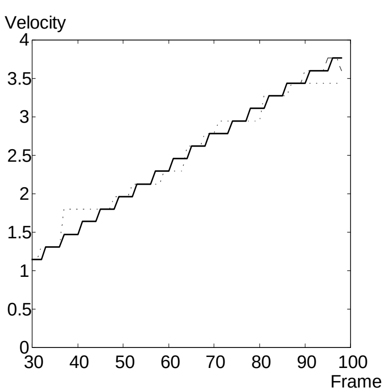

Figure 3: Velocity estimation based on: Spectrogram (dotted line), Wigner distribution (dashed line) and local polynomial periodogram (solid line).

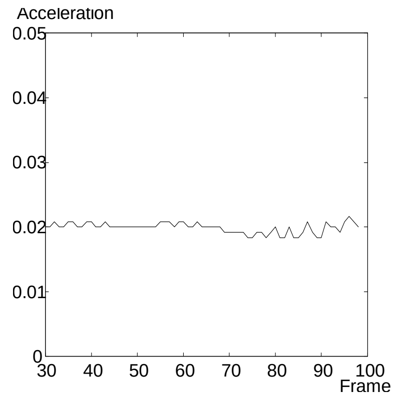

Figure 4: Acceleration estimation based on local polynomial periodogram.

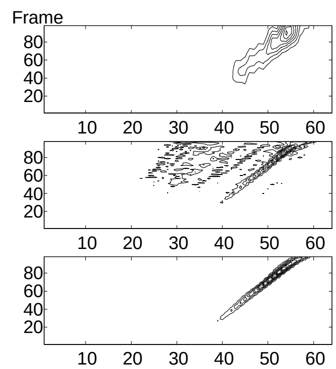

Figure 5: Time-frequency representation - noisy case; a) Spectrogram; b) Wigner distribution c) Local polynomial periodogram.

estimation based on the spectrogram, Wigner distribution and the local polynomial periodogram is shown. It is shown that the local polynomial periodogram estimates the velocity accurately, even in this case. In addition, it provides the acceleration estimation, Fig.7.

5. CONCLUSION

Time-varying velocity and acceleration estimation has been performed by using the local polynomial periodogram. This approach does not require modification in the SLIDE algorithm. Advantages of this approach are confirmed by numerical examples. Application of the proposed approach to real video-sequence and comparison with other existing methods [6] will be further step in our research.

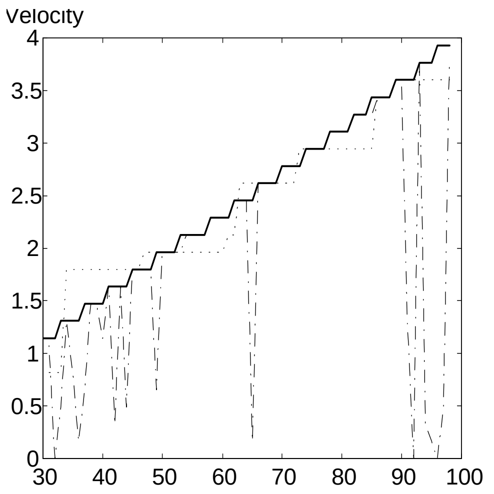

Figure 6: Velocity estimation in the noisy case, based on: Spectrogram (dotted line); Wigner distribution (dashed line) and local polynomial periodogram (solid line).

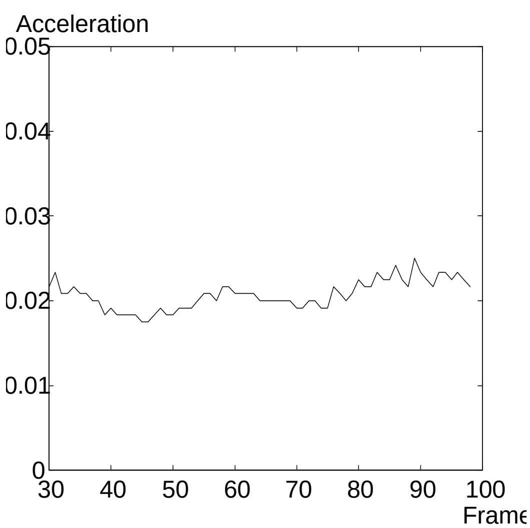

Figure 7: Acceleration estimation based on the local polynomial periodogram (noisy case).

6. ACKNOWLEDGMENT

This work is supported by the Volkswagen Stiftung, Federal Republic of Germany.

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