On the Space-Varying Filtering

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Abstract

Filtering of two-dimensional noisy signals is considered in the paper. Concept of nonstationary space-varying filtering is used. It has been shown that the space-varying filtering, along with the algorithm for adaptive Wigner distribution estimation, may be efficiently used in filtering of two-dimensional signals with high amount of noise, when the noise overlap with signal in space and frequency separately, but not significantly in the joint space/spatial-frequency domain.

I. INTRODUCTION

When a two-dimensional signal and noise do not occupy the same frequency range then efficient filtering can be performed using stationary filters. But in the cases when the signal and noise overlap in a significant part of the space and frequencies then the stationary filtering may be difficult and inefficient. We will consider signals and noises that occupy the same space and spatial-frequency domain, but when their separation may be done in the joint space/spatial-frequency domain. For this kind of noisy signals a concept of space-varying filters is presented. It is an extension of the one-dimensional time-varying filtering approach to the two-dimensional problems [3], [10], [13] using only one noisy signal realization. Since the concept of the time-varying filtering is based on the time-frequency distributions, we will here use joint space/spatial-frequency distributions, [5], [8], [12], [17], [18], [19], in order to define and implement space-varying filtering. The two-dimensional Wigner distribution, along with the two-dimensional Weyl operator, is used as a basic distribution. An algorithm for the estimation of the Wigner distribution of signal using only one noisy observation is presented. The result is applied to the space-varying filtering of a signal with a high amount of noise. *The implementation is based on a single noisy signal realization.*

The paper is organized as follows. The concept of space-varying filtering is presented next. The pseudo forms of the filtering relations are introduced. Based on a specific statistical approach of comparing the bias and the variance, an algorithm for the Wigner distribution estimation with minimal mean square error is presented in Section III. The algorithm and space-varying filtering efficiency is demonstrated in Section IV on examples with monocomponent, multicomponent and real image signals.

II. THEORY

Consider a two-dimensional noisy signal

$$s(x,y)=f(x,y)+\nu(x,y), \tag{1}$$

where $f(x,y)$ is the signal, while $\nu(x,y)$ denotes the noise. It will be assumed that noise is white, Gaussian, with indepentent real and imaginary parts, where noise variance is σ_ν^2. The above relation may be written in a vector notation as

$$s(\vec{r})=f(\vec{r})+\nu(\vec{r}) \text{ where } \vec{r}=(x,y). \tag{2}$$

Nonstationary two-dimensional filtering will be defined in analogy with the one-dimensional time-varying filtering

$$(Hs)(\vec{r})=\int_{\vec{v}} h(\vec{r}+\vec{v}/2,\vec{r}-\vec{v}/2)s(\vec{r}+\vec{v})d\vec{v} \tag{3}$$

where $h(\vec{r},\vec{v})$ is the impulse response of the space-varying two-dimensional filter. This form of filtering definition provides that for $f(\vec{r})=Ae^{j\phi(\vec{r})}$ we get $(Hf)(\vec{r})=cf(\vec{r})$ if the filter in space/spatial-frequency domain is defined as a delta function $\delta(\vec{\omega}-\nabla\phi(\vec{r}))$ along the local frequency $\nabla\phi(\vec{r})$, for signals that satisfy stationary phase method conditions [1], [11].

The optimal value of H will be derived by analogy with the Wiener filter in the stationary signal cases. When the mean square error $e^2=E\{|f(\vec{r})-(Hs)(\vec{r})|^2\}$ reaches its minimum, the error $f(\vec{r})-(Hs)(\vec{r})$ is orthogonal to the data $s^*(\vec{r}+\vec{\alpha})$. From this fact we get

$$E\{[f(\vec{r})-\int_{\vec{v}} h(\vec{r}+\vec{v}/2,\vec{r}-\vec{v}/2)s(\vec{r}+\vec{v})d\vec{v}]s^*(\vec{r}+\vec{\alpha})\}=0 \tag{4}$$

From (4) follows

$$\overline{AF}_{fs}(\vec{\theta},\vec{\alpha})=\int_{\vec{v}}\int_{\vec{u}} A_H(\vec{u},-\vec{v})\overline{AF}_{ss}(\vec{\theta}-\vec{u},\vec{\alpha}-\vec{v})d\vec{v}d\vec{u} \tag{5}$$

for the underspread processes when $e^{j(\vec{\theta}\vec{v}-\vec{u}\vec{\alpha}-\vec{u}\vec{r})/2} \cong 1$. The expected value of ambiguity function is defined as

$$\overline{AF}_{ss}(\vec{\theta},\vec{v}) = \int_{\vec{r}} E\{s(\vec{r}+\vec{v}/2)s^*(\vec{r}-\vec{v}/2)\}e^{-j\vec{\theta}\vec{r}}d\vec{r} \tag{6}$$

and

$$\overline{A}_H(\vec{\theta},\vec{v}) = \int_{\vec{r}} h(\vec{r}+\vec{v}/2,\vec{r}-\vec{v}/2)\}e^{-j\vec{\theta}\vec{r}}d\vec{r} \tag{7}$$

Taking two-dimensional Fourier transform of (5) we get

$$\overline{WD}_{fs}(\vec{r},\vec{\omega}) = L_H(\vec{r},\vec{\omega})\overline{WD}_{ss}(\vec{r},\vec{\omega}) \tag{8}$$

where $\overline{WD}_{ss}(\vec{r},\vec{\omega})$ is the Wigner spectrum (the expected value of the Wigner distribution) of signal $s(\vec{r})$

$$\overline{WD}_{ss}(\vec{r},\vec{\omega}) = \int_{\vec{v}} E\{s(\vec{r}+\vec{v}/2)s^*(\vec{r}-\vec{v}/2)\}e^{-j\vec{\omega}\vec{v}}d\vec{v} \tag{9}$$

and $L_H(\vec{r},\vec{\omega})$ is Weyl symbol of the filter impulse response denoted by

$$L_H(\vec{r},\vec{\omega}) = \int_{\vec{v}} h(\vec{r}-\frac{\vec{v}}{2},\vec{r}+\frac{\vec{v}}{2})e^{-j\vec{\omega}\vec{v}}d\vec{v}. \tag{10}$$

For the signal not correlated with noise follows

$$L_H(\vec{r},\vec{\omega}) = \frac{\overline{WD}_{ff}(\vec{r},\vec{\omega})}{\overline{WD}_{ff}(\vec{r},\vec{\omega}) + \overline{WD}_{\nu\nu}(\vec{r},\vec{\omega})} \tag{11}$$

Suppose that the Wigner spectrum of the random signal $f(\vec{r})$ lies inside a region R_f in the space/spatial-frequency plane, while the noise lies outside this area, except may be its small part that can be neglected with respect to the part of noise outside R_f. This is true, for example, for a wide class of frequency modulated (highly concentrated in the space/spatial-frequency plane) signals $f(\vec{r})$, corrupted with a white noise $\nu(\vec{r})$, widely spread in the space/spatial-frequency plane. A simple solution satisfying these requirements is given by

$$L_H(\vec{r},\vec{\omega}) = \begin{cases} 1 & \text{for } (\vec{r},\vec{\omega}) \in R_f \\ 0 & \text{for } (\vec{r},\vec{\omega}) \notin R_f \end{cases} \tag{12}$$

where R_f is the region where $\overline{WD}_f(\vec{r},\vec{\omega}) \neq 0$. In practice this situation appears in the cases of frequency modulated signals, for example in optics, when the signals are well concentrated in the joint space-frequency domain, along arbitrary lines, while the noise is widely spread over entire frequency domain. But in this case we should also know, not the distribution itself, but its region of support R_f. This makes the problem easier, but does not overcome the basic difficulty of knowing the distribution support of signal without noise. An efficient determination of the support R_f will be the topic in the sequel, after some practical hints on the filtering realization are given.

In the numerical implementations the pseudo (space limited) formes of the filtering relations should be introduced. The pseudo form of operator (3) will be defined by

$$(Hs)(\vec{r}) = \int\limits_{\vec{v}} h(\vec{r}+\vec{v}/2,\vec{r}-\vec{v}/2)w(\vec{v})s(\vec{r}+\vec{v})d\vec{v} \tag{13}$$

This relation enables the space limited intervals to be used. It may be shown that for the case of signals ideally concentrated along the local frequency in the space-frequency domain the following important conclusion holds: The window $w(\vec{v})$ does not influence the filter output (13) as far as $w(\vec{0}) = const = 1$. Using the Parseval's theorem relation (13) assumes the form

$$(Hs)(\vec{r}) = \frac{1}{4\pi^2}\int\limits_{\vec{\omega}} L_H(\vec{r},\vec{\omega})STFT(\vec{r},\vec{\omega})d\vec{\omega} \tag{14}$$

where $STFT(\vec{r},\vec{\omega})$ is the "short space" Fourier transform defined as

$$STFT(\vec{r},\vec{\omega}) = \int\limits_{\vec{v}} s(\vec{r}+\vec{v})w(\vec{v})e^{-j\vec{\omega}\vec{v}}d\vec{v} \tag{15}$$

Therefore, in order to perform a two-dimensional space-varying filtering we should know $STFT$ and L_H. Calculation of $STFT$ is simple, but the problem of L_H determination still remains. Obviously a precise determination of L_H is directly related to precise R_f determination, what further leads to *the estimation of the Wigner distribution of signal without noise, using only one noisy signal observation, with the mean square error as small as possible.* Note that the determination of the support region based on the squared modulus of $STFT(\vec{r},\vec{\omega})$, i.e., multidimensional spectrogram, would be appropriate only in the case when the local frequency does not change over space or changes very slowly so that it may be considered as a constant within the window $w(\vec{v})$.

Fig. 1. Determination of the region of support R_f for the space-varying filter at the point $(x, y) = (0.5, 0.5)$: a) Wigner distribution of noisy signal calculated using the window $N_2 \times N_2 = 128 \times 128$, b) Wigner distribution of the noisy signal calculated using the proposed algorithm. Distribution from (b) is used for R_f determination at the point $(x, y) = (0.5, 0.5)$.

III. ALGORITHM

Here, it we will be presented algorithm for Wigner distribution estimation using only two window widths in calculations. Details on derivations may be found in [16]. Denote these two window widths by $N_1 \times N_1$ and $N_2 \times N_2$ where $N_1 \ll N_2$. Window $N_1 \times N_1$ produces small variance, while $N_2 \times N_2$ has small bias. Therefore, when the confidence intervals for these two windows intersect then it means that the bias is small and we use $N_1 \times N_1$ in order to reduce the variance. Otherwise, the bias is large and we use $N_2 \times N_2$ in order to reduce it. The resulting adaptive Wigner distribution is

$$WD_s^e(\vec{n}, \vec{\omega}) = \begin{cases} WD_s(\vec{n}, \vec{\omega}; N_1) & \Phi \;\; true \\ WD_s(\vec{n}, \vec{\omega}; N_2) & elsewhere \end{cases} \tag{16}$$

where for $\Phi\, true$ holds

$$|WD_s(\vec{n}, \vec{\omega}; N_1) - WD_s(\vec{n}, \vec{\omega}; N_2)| \leq (\kappa + \triangle\kappa)[\sigma_s(N_1) + \sigma_s(N_2)] \tag{17}$$

In this way using only two window widths we may expect significant improvements, since the Wigner distribution is either slow-varying (bias very small) or highly concentrated along the instantaneous frequency (bias very large). If we used a multi-window approach with many windows between $N_1 \times N_1$ and $N_2 \times N_2$ then the algorithm would be dominantly selecting these two extreme window widths.

Relation (16) reduces to the calculation of the Wigner distribution and its variance for two-window widths. One possible relation for the variance estimation is

$$\sigma_{ss}^2(N) \approx \frac{9N^2}{64} \left(\sum_{m_1=-N/2}^{N/2-1} \sum_{m_2=-N/2}^{N/2-1} |s(\vec{n} + \vec{m}|^2 \right)^2 \tag{18}$$

It holds for the low signal to noise ratio. For cases of small noise the variance estimation procedure is given in [**?**]. From (18) follows that $\sigma_{ss}^2(N_1) = \sigma_{ss}^2(N_2)N_1^2/N_2^2$. Thus, we have defined the algorithm and all parameters for the Wigner distribution calculation, which is then used for the region of support R_f determination and space varying filtering.

For multicomponent signal case $f(\vec{n}) = \sum_{i=1}^{M} f_i(\vec{n})$ the WD contains M-signal terms (auto-terms) and $M(M-1)/2$ interference terms (cross-terms) with amplitude that could cover the signal terms

$$WD_f(\vec{n}, \vec{\omega}) = \sum_{i=1}^{M} WD_{ii}(\vec{n}, \vec{\omega}) + 2\,\mathrm{Re} \sum_{i=1}^{M} \sum_{j>i}^{M} WD_{ij}(\vec{n}, \vec{\omega}) \tag{19}$$

This was the reason why the RID class of distributions [9] has been introduced

$$CD_f(\vec{n}, \vec{\omega}) = WD_f(\vec{n}, \vec{\omega}) *_{\vec{n}} *_{\vec{\omega}} \Pi(\vec{n}, \vec{\omega}) \tag{20}$$

Fig. 2. Filtering of a twodimensional signal: a) Original signal without noise, b) Noisy signal, c) Noisy signal filtered using the stationary filter with a cutoff frequency equal to the maximal signal frequency, d) Noisy signal filtered using the stationary filter with a cutoff frequency equal to a half of the maximal frequency, e) Noisy signal filtered using the stationary filter with a cutoff frequency equal to a fourth of the maximal frequency, f) Noisy signal filtered using the space-varying filter.

where $\Pi(\vec{n},\vec{\omega})$ [4] is the kernel in space/spatial-frequency plane, whose Fourier transform $c(\vec{\theta},\vec{\tau}) = FT\{\Pi(\vec{n},\vec{\omega})\}$ (kernel in ambiguity plane) is a low-pass filter function. For multicomponent signals we will use the S-method (SM) [14], [17]:

$$S(\vec{n},\vec{\omega}) = \sum_{\vec{\theta}} P(\vec{\theta}) STFT(\vec{n},\vec{\omega}+\vec{\theta}) STFT^{*}(\vec{n},\vec{\omega}-\vec{\theta}) \tag{21}$$

where $P(\vec{\theta})$ is rectangular window with $2L+1$ width in each directions. The SM belongs to general Cohen class of distribution [4]. When the components of a multicomponent signal do not overlap in the joint space/spatial-frequency plane, it is possible to determine the width of the frequency window $P(\vec{\theta})$ so that the SM is equal to a sum of the WDs os each signal component individually (sum of the auto-terms) [14]

$$S(\vec{n},\vec{\omega}) = \sum_{i=1}^{M} WD_{ii}(\vec{n},\vec{\omega}) \tag{22}$$

This interesting property has attracted the attention of some other researches to use the SM in their work [7], [6], [2]. Noise influence on the SM was considered in [15]. The variance for nonoverlapped multicomponent signals can here be expressed as

$$\sigma_{ss}^{2}(\vec{n},\vec{\omega}) = \begin{cases} \sigma_{\nu}^{2} \sum\limits_{\vec{m}} w^{4}(\vec{m})[2A_{i}^{2}(\vec{n}) + \sigma_{\nu}^{2}] & for\ (\vec{n},\vec{\omega}) \in R_{f_i} \\ \sigma_{\nu}^{4} \sum\limits_{\vec{m}} w^{4}(\vec{m}) & for\ (\vec{n},\vec{\omega}) \notin R_{f_i},\ i=1,..,M \end{cases} \tag{23}$$

The estimation of sum the auto-terms in the WD, based on the SM, could be written as:

$$SM^{e}(\vec{n},\vec{\omega}) = \begin{cases} SM(\vec{n},\vec{\omega}; N_1, L=0) & \Phi\ \ true \\ SM(\vec{n},\vec{\omega}; N_2, L) & elsewhere \end{cases} \tag{24}$$

where for $\Phi\, true$ holds

$$|SM(\vec{n},\vec{\omega};N_1,L=0)-SM(\vec{n},\vec{\omega};N_2,L)|\leq(\kappa+\triangle\kappa)[\sigma_{ss}(N_1,L=0)+\sigma_{ss}(N_2,L)] \tag{25}$$

Note that for $L=0$ the spectrogram (square magnitude of the STFT) is obtained as a special cases of the SM. Region of signals support R_f could be obtained from $SM^e(\vec{n},\vec{\omega})$, as

$$L_H(\vec{n},\vec{\omega})=\left[\frac{1}{2}+\frac{1}{2}sign(SM^e(\vec{n},\vec{\omega})-Lev)\right] \tag{26}$$

where Lev is a reference level given for our numerical calculations. Here we used $Lev=0.1\max_{\vec{n}}(SM^e(\vec{n},\vec{\omega}))$.

IV. NUMERICAL EXAMPLES

Example 1. The presented theory will be illustrated on the numerical example with the signal

$$f(x,y)=0.5\exp(j(96\pi x^2+96\pi y^2))\exp(-1.5x^2-1.5y^2) \tag{27}$$

corrupted with a high amount of additive noise with variance $\sigma^2_{\nu\nu}=1$, meaning that $10\log(A^2/\ \sigma^2_{\nu\nu})\leq-6[dB]$. The signal form (27) is inspired with the interferograms in optics, where it appears.

Figure 1 demonstrates the algorithm for the region R_f determination, at the point $(x,y)=(0.5\,,0.5\,)$. The Wigner distribution calculated using the window width $(N_2\times N_2)=(128\times128)$ is shown in Figure 1a. Figure 1b presents the Wigner distribution calculated using the two-window algorithm presented in Section IV with $(N_1\times N_1)=(16\times16)$ and $(N_2\times N_2)=(128\times128)$. The algorithm used the lower variance distribution calculated with $(N_1\times N_1)=(16\times16)$ everywhere, except at the point where the signal energy is concentrated, where the lower bias window $(N_2\times N_2)=(128\times128)$ is used.

Noisy signal (27) is considered within the interval $[0\leq x<0.5,0\leq y<0.5]$. Signal frequency changes along both axis from 0 to $(3/4)f_{\max}$ where $f_{\max}$ is the maximal frequency for a given sampling interval. In (17) we assumed $\kappa+\triangle\kappa=3.5$.

Original signal without noise is shown in Figure 2a, while noisy signal is given in Figure 2b. Signal filtered using stationary filters with the cutoff frequency in both directions $f_c=(3/4)f_{\max}$ is given in Figure 2c. Filtering with lower cutoff frequencies reduces the noise, but also degrades the signal, Figures 2d, 2e with $f_c=f_{\max}/2$ and $f_c=f_{\max}/4$, respectively. Signal obtained from the noisy signal, Figure 2b, using the space-varying filtering presented in this paper, by (3),(14) and the algorithm in Sec.IV, is shown in Figure 2f. The advantage of the proposed concept with respect to the stationary filtering is evident.

Example 2. For multicomponent case we will, as an example, consider the signal:

$$f(x,y)=\exp(j(-104\pi(x^2+x)-104\pi y^2))+\exp(j(-104\pi x^2-104\pi(y^2+y))) \tag{28}$$

corrupted with a high amount of additive noise with variance $\sigma^2_{\nu\nu}=1.7$. In the spectrogram calculation the Hanning window with $(N_1\times N_1)=(16\times16)$ is used, while for the SM calculation Hanning window with $(N_2\times N_2)=(128\times128)$ and $L=2$, are taken. Original signal is shown in Figure 3a. Noisy signal is given in Figure 3b. Signal filtered with the proposed algorithm is presented in Figure 3c.

Fig. 3. a) Two-dimensional multicomponent signal without noise, b) Noisy two-dimensional multicomponent signal, c) Filtered noisy signal.

Example 3. Finally, in the last example we will consider a separation of linear frequency modulated signals from real image. This would correspond to a generalization of the notch filtering in the stationary cases. Here the local frequency is varying and the position of the notch frequency changes for each point. Its detection based on the spectrogram would not be precise what would cause an uprecise and wide support region, resulting in unsatisfactory separation. The spectrogram based region of support determination would be efficient only in the cases of constant (or slow-varying) local frequency. The Wigner distribution approach gives very precise determination of the local frequency when it changes over image. It results in an efficient separation using the proposed procedure, as demonstrated in Fig.4. Figure 4a, shows original image. A part of the frequency modulated signal added to the image Fig.4a is shown in Fig.4b. Original figure corrupted with the frequency modulated signal is shown in Fig.4c, while the frequency modulated signal extracted from Fig.4c, using the proposed procedure is shown in Fig.4d. The reconstruction is very good, although the ratio of the original image maximal value to the frequency modulated signal maximal value was extremely low,

$$20\log_{10}\frac{\max\{\nu(\vec{r})\}}{\max\{f(\vec{r})\}} = -18.5[dB], \tag{29}$$

where $\nu(\vec{r})$ is the image and $f(\vec{r})$ denotes frequency modulated signal added to image.

Fig. 4. a) The original image; b) A part of the linear frequency modulated signal added to the image; c) A sum of the previous two signals; d) Reconstructed linear frequency modulated signal from figure c).

V. CONCLUSION

The concept of space-varying filtering in multidimensional signals is presented. Its efficiency is illustrated on a noisy signal whose form is inspired by the interferograms in optics.

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