Combined TV filtering method and CS signal reconstruction

Marijana Šćekić, Radomir Mihajlović, Andjela Draganić and Irena Orović, *Member*, IEEE

***Abstract*— The procedure that combines Total Variation filtering method and Compressive Sensing signal reconstruction is proposed in this paper. Recently, Compressive Sensing has been intensively studied as a method for signals acquisition. It has been shown that signals can be reconstructed by using just a small set of random samples. However, the signal reconstruction may not be efficient in the presence of noise. Therefore, we considered a combined approach that performs Total Variation filtering prior to Compressive Sensing reconstruction, in order to provide high accuracy of reconstruction results. The procedure is tested on signals that appear in wireless communications. The experiments demonstrate that the Total Variation procedure successfully eliminates the Gaussian noise, while the filtered signal can be successfully recovered using only 30% of signal samples.**

***Index Terms*—compressive sensing, total variation, l_1 minimization, wireless signals**

I. INTRODUCTION

According to the Shannon-Nyquist sampling theorem, signal should be sampled at the rate that is at least twice higher than the maximal signal frequency, in order to preserve important information about the signal. Wide-band signals, sampled in this way, produce large number of samples. Therefore, in order to reduce memory requirements and to lower the acquisition time, different methods for Compressive Sensing (CS) approach have been developed [1]-[7].

CS assumes that a signal can be recovered using small number of randomly chosen samples, as long as the signal satisfies some a priori defined conditions, such as sparsity and incoherence property. Sparsity is related to the property that the signal can be represented by a small number of non-zero coefficients in a certain transform domain. Most of the existing compression algorithms are also based on the sparsity property. Furthermore, the acquired signal samples, i.e., measurements should be incoherent. Incoherence provides CS reconstruction with a quite small number of samples. The reconstruction of the signal is based on the optimization algorithms [7]-[11], usually on the l_1 norm minimization, or approximate solutions based on the greedy algorithms.

Generally speaking, the noisy signals cannot be accurately reconstructed using the standard optimization algorithms, and thus, the solution should be based on the combined denoising and reconstruction methods. In this paper, the signal denoising is based on the Total Variation minimization (TV) algorithm [12]-[14], which is applied to signals in the Gaussian noise environment. After denoising, the CS procedure is applied to provide a small set of measurements, which will afterwards allow an accurate reconstruction of the entire signal. Therefore, the goal of using CS is to significantly decrease the number of samples required to represent the considered signals (e.g. for the purpose of transmission). It has been experimentally proven that the exact signal reconstruction can be achieved.

The paper is organized as follows. The basic theory related to the TV denoising and CS reconstruction is given in Section II. Section III contains analysis of TV denoising and CS reconstruction procedure applied to wireless signals. Section IV contains experimental results and error analysis. Conclusion is given in Section V.

II. THEORETICAL BACKGROUND

A. Total Variation

Total variation (TV) [12]-[14] has been introduced as denoising technique for 2D signals first. Lately, TV is used in CS, interpolation and in-painting, as well. It can be applied both for the 1D and 2D signal filtering and restoration. The TV of the discrete signal x is defined as follows:

$$TV(x)=\sum_{i,j}\sqrt{(x_{i+1,j}-x_{i,j})^2+(x_{i,j+1}-x_{i,j})^2}\,. \quad (1)$$

In the case of noisy signal $x_n = x+n$, estimation of the signal x could be performed by minimizing:

$$\min_x \|x_n - x\|_{l_2}^2 + \lambda\|Ax\|_{l_1}\,, \quad (2)$$

where A is matrix defined as:

$$A=\begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & ... & 1 & \\ & & & -1 & \end{bmatrix},$$

and λ is smoothing parameter. Relation (2) is called TV denoising. Parameter λ is regularization parameter and

All authors are with the Faculty of Electrical Engineering, University of Montenegro, Džordža Vašingtona bb. 81000 Podgorica, Montenegro (e-mails: scekic.marijana@hotmail.com, andjelad@ac.me, irenao@ac.me).

controls the amount of smoothing. Larger values of λ are used to remove higher noise. In this paper, dual formulation for the l_1 norm of the x is used: $|x|_{l_1} = \max_{|p|\leq 1} p^t x$, as it is easily performed. Then, the minimization problem becomes [14]:

$$\min_x \max_{|p|\leq 1} \|x_n - x\|_{l_2}^2 + \lambda p^t Ax. \tag{3}$$

B. Compressive Sensing

Efficient signal analysis using traditional sampling technique, requires a large set of signal samples. Consequently, further signal processing becomes demanding. CS provides signal analysis using significantly less samples than it is required by the Sampling Theorem. CS enables signal reconstruction from small number of samples, acquired in a random manner. In order to properly reconstruct the signal form its measurements, the measurement procedure should be incoherent and the signal should be sparse. Sparsity means that the signal, having a dense representation in one domain, could be represented with small number of non-zero coefficients in the other (sparse) domain. The domain of signal sparsity could be the Discrete Fourier Transform (DFT) domain, Discrete Cosine Transform (DCT) domain, wavelet domain, etc.

Let us describe CS method on the discrete-time signal x. Signal consists of the N samples in the time domain, and K non-zero samples in the frequency domain. Therefore, it is said that the signal is K-sparse. Signal can be represented in terms of basis vectors as follows [15], [16]:

$$x = \sum_{i=1}^{N} w_i \psi_i = \psi w, \tag{4}$$

where w_i is weighting coefficient, ψ_i is basis vector, ψ denotes NxN transform matrix whose columns are basis vectors and w is the equivalent of the signal in ψ domain. Sparse signal domain has its dense domain equivalent, from which the samples are acquired. The number of measurements M is larger than the number of non-zero components K, but is significantly lower than the length of the signal. Acquisition of the samples is performed using the measurement matrix ϕ. If we denote measurement vector as y, then the equation:

$$y_{M\text{x}1} = \phi_{M\text{x}N} x_{N\text{x}1}, \tag{5}$$

holds, i.e.:

$$y = \phi x = \phi \psi w = \xi w, \tag{6}$$

where ξ denotes CS matrix. As (6) has M equations with N unknowns, this system of equations is undetermined ($M < N$) and has infinite number of solutions. To solve this problem, optimization algorithms are used, such as primal-dual interior point method, greedy methods such as orthogonal matching pursuit, non-iterative and iterative variance based algorithms [7]-[11], [17], [18], etc.

Commonly used optimization technique is based on the l_1 norm minimization. The optimization problem is defined as:

$$\min \|x\|_{l_1} \ \textit{subject to} \ y = \xi x. \tag{7}$$

Vector obtained by solving (7) will have the sparsest representation among infinite number of possible solutions.

III. Combined CS and TV procedure for wireless signal reconstruction

In this paper, signals that appear in wireless communications are used to test combined TV and CS reconstruction procedure. Wireless signals could have wide frequency band, and therefore require large number of samples to be analyzed [19], [20]. As wireless signals could be observed as a sum of the small number of sinusoids, the sparsity property will be satisfied in the Fourier domain.

In this paper we have observed the signal, defined as:

$$x_n(t) = \sum_{i=1}^{K} \sin(2\pi f_i t) + e, \tag{8}$$

where e is additive Gaussian noise. For the signal reconstruction we have used l_1 minimization primal dual interior point method. As this method shows poor results in the noisy signal case, the TV filtering is firstly performed. The algorithm could be summarized as follows:

1. Having noisy signal x_n and defining smoothing parameter λ, form the optimization function F as follows:

$$F = \min_x \max_{|p|\leq 1} \|x_n - x\|_{l_2}^2 + \lambda p^t Ax. \tag{9}$$

2. Finding vectors x and p is based on iterative procedure, i. e.

$$\begin{aligned} x^{(i+1)} &= y - \frac{\lambda}{2} A^t p^{(i)}, \\ p^{(i+1)} &= C(p^{(i)} + \frac{2}{\alpha\lambda} Ax^{(i+1)}, 1) \end{aligned}. \tag{10}$$

Number of iterations, as well as parameter λ is user defined. Parameter α is chosen to be larger than the maximum eigenvalue of AA^t and it is sufficient to use $\alpha>2$. Operator C is the clipping operator, and it is defined as:

$$C(c,1) = \begin{cases} c, & |c| \leq 1 \\ 1 \times sign(c), & |c| \geq 1 \end{cases}.$$

3. After denoising, signal can be reconstructed using primal dual interior point method. DFT matrix ψ is used as a basis matrix. Measurements are obtained by using set of rows from

matrix ψ. Random permutations of samples positions are stored in vector d and they determine M available signal samples, i.e. M matrix rows:

$$d = P(N), \tag{11}$$

where P denotes random permutations of N elements and N is the signal length. CS matrix ξ is then obtained as:

$$\zeta = \psi(1:d(1:M),1:N)\,. \tag{12}$$

The optimization problem is solved with l_1 minimization (from the L1-Magic Matlab toolbox). The experiments are performed under presence of Gaussian noise of different strength (different signal to noise ratio – SNR). As l_1 minimization algorithm does not give satisfactory results in the noisy case, the TV denoising procedure is firstly applied. Afterwards, it has been shown that the signal can be completely recovered from small set of noise-free signal samples.

In the experiments we have repeated the TV denoising algorithm more than once, in order to eliminate the noise as much as possible. TV algorithm is based on signal smoothing operations. Therefore, the smoothing parameter λ should be carefully chosen according to the number of iterations and the amount of noise.

IV. EXPERIMENTAL RESULTS

In the sequel we provide the experimental results for denoising and reconstruction of sparse signals.

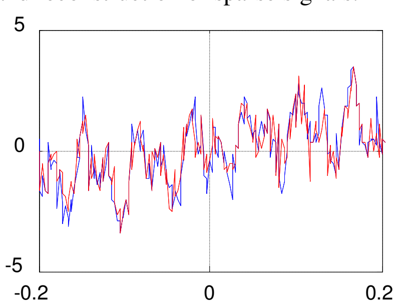

Figure 1. Time domain of noisy (blue) and reconstructed (red) signal

The test signal consists of three sinusoidal components and it is corrupted by Gaussian noise. The number of components in the signal is K=3, with frequencies f_1=2, f_2=-16, f_3=32, while *t=-1:1/512:1-1/512.* The results shown in the following figures are obtained for the SNR=6.5052 dB. The noisy signal is presented in the Figure 1 (blue line), while its reconstructed version obtained using 30% of noisy samples is plotted with red line. As it can be seen from Figure 1, l_1 minimization algorithm fails to reconstruct the signal, due to the presence of noise which destroys signal sparsity. Therefore, signal is first filtered using TV method.

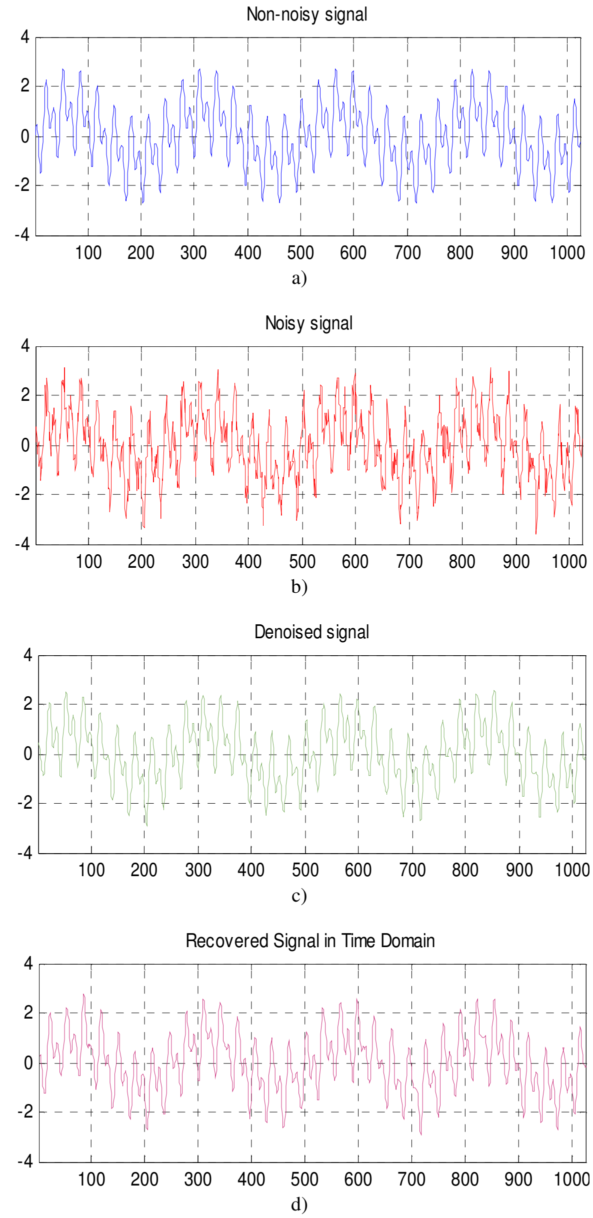

Figure 2. Time domain of: a) original (non-noisy signal); b) signal corrupted by Gaussian noise; c) signal denoised using TV algorithm; d) signal recovered from 30% of non-noisy measurements

TV denoising is performed with smoothing parameter $\lambda = 4$. Denoising is performed twice, in order to minimize the error between original (noise-free) and denoised signal. In Figure 2, the original, noisy and denoised signals are shown (Figure 2a, b and c, respectively). Figure 2d shows the reconstructed signal using only 30% of samples. In Figure 3, the zoomed regions of the original, noisy, denoised and reconstructed signals are illustrated, as well.

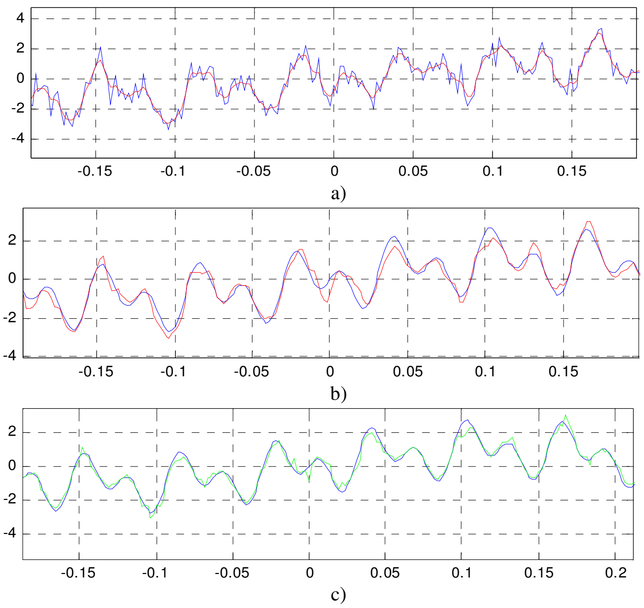

Figure 3. Time domain of: a) noisy (blue) and denoised (red) signal; b) non-noisy (blue) and denoised (red) signal; c) non-noisy (blue) and reconstructed (green) signal

The Fourier transforms of non-noisy, noisy and denoised signal are presented in Figure 4. The denoised and reconstructed signal using 30% of signal samples is presented in the Figure 4c (green), together with the original signal (blue). The difference between original and reconstructed signal is small, as it can be seen from the Figure 4c. Denoising and CS reconstruction accuracy is verified numerically by calculating errors - mean square and mean absolute errors between the signals. Table I shows mean absolute error between original and denoised signal (MAE_OD), mean square error between original and denoised signal (MSE_OD), as well as the mean absolute and mean square error between denoised and reconstructed signals (MAE_DR, MSE_DR). The calculated errors are small, compared to the mean square value of the signal (which is equal to 1.5) or mean absolute signal value (which is equal to 1.0086).

TABLE I

MAE AND MSE BETWEEN ORIGINAL AND DENOISED SIGNAL AND BETWEEN DENOISED AND RECONSTRUCTED SIGNAL

SNR [dB]	MAE_OD	MAE_DR	MSE_OD	MSE_DR
2.9771	0.3815	0.3123	0.2297	0.2172
4.9669	0.3121	0.3263	0.1507	0.2498
6.5052	0.2449	0.1768	0.0927	0.0685
7.9428	0.2110	0.1693	0.0692	0.0632
12.8014	0.1325	0.0914	0.0277	0.0184

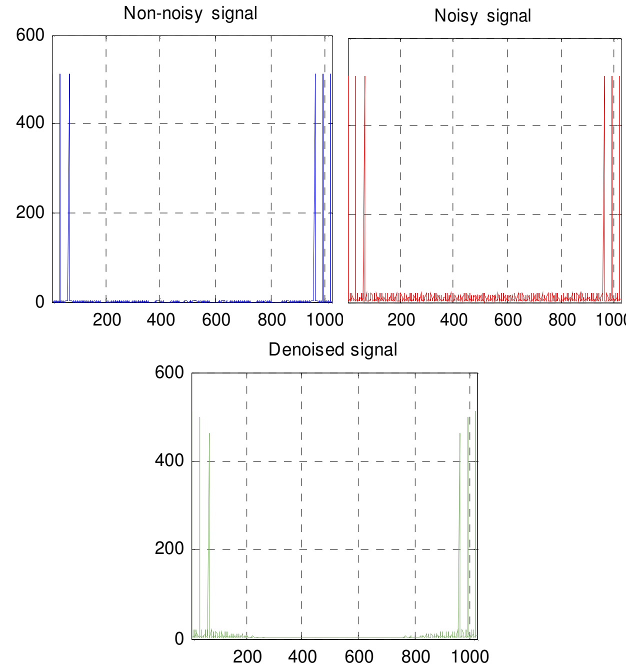

Figure 4. Fourier transform of non-noisy (blue), signal corrupted by impulse noise (red) and denoised signal (green)

V. CONCLUSION

The combined total variation denoising and compressive sensing reconstruction procedure is proposed for the case of noisy signals in wireless communications. The l_1-norm based optimization technique for CS reconstruction can be applied only after the signal denoising, which is performed by using TV filtering method. The signals corrupted by Gaussian noise are observed in the paper. It has been shown that, by using TV filtering, much of the noise can be removed. This is proved by calculating errors between the non-noisy and the filtered signal. Finally, after signal denoising, the CS can be applied assuring high reconstruction accuracy using just 30% of the signal samples.

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