A Virtual Instrument for Compressive Sensing of Multimedia Signals

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Abstract— **Compressive Sensing (CS) is currently a very popular signal acquisition approach. It provides an alternative way of signal sampling which is based on a small random set of measurements. The entire signal can be reconstructed from the measurements with high accuracy by using very complex mathematical algorithms if the certain conditions are met. Various algorithms for CS reconstruction have been proposed for different types of signals and different application requirements. In this paper, several commonly used algorithms for one-dimensional (1D) and two-dimensional (2D) signals reconstruction are implemented within the Virtual Instrument for CS signals reconstruction. The Virtual Instrument is a user-friendly tool that provides efficient analysis of signals, using different algorithms and variety of options and parameters. It includes different multimedia test signals (both 1D and 2D signals), but also there is an option for user-defined signals.**

Keywords – Compressive Sensing, reconstruction, l_1-norm minimization, Orthogonal Matching Pursuit, TV minimization

I. INTRODUCTION

Compressive Sensing/Sampling (CS) [1]-[4] is a recently developed approach for signal sampling and reconstruction. It allows a signal, having a sparse representation in certain domain, to be recovered from a small set of randomly acquired samples. Signal exhibits sparsity property if it can be represented by a small number of non-zero samples in the certain transform domain. The signal recovering procedure is based on minimization of different norms (l_0, l_1, l_2 norm, etc.) [1]. The commonly used approach is based on l_1-norm minimization [5]-[7], solved by convex optimization algorithms. Another type of algorithms, called greedy algorithms (e.g. Orthogonal Matching Pursuit OMP, Gradient Pursuit GP [8]-[10], etc.) are based on iterative procedures. They provide much simpler and faster solutions, but may require some a priori knowledge about the signals. Generally speaking, these algorithms may provide different results depending on the signal nature, number of available measurements, or signal sparsity.

In this paper, the Virtual Instrument [11] for CS signal reconstruction is presented. Several algorithms for one-dimensional (1D) and two-dimensional (2D) multimedia data reconstruction are implemented. Virtual Instrument allows user to choose between 1D and 2D signal reconstruction, as well as to test different algorithms and compare their performance. Additionally, the users may change the number of measurements and the domain of sparsity used in CS reconstruction. The reconstruction accuracy is quantified using different error types.

The paper is organized as follows. In Section II, the theoretical background on CS theory and reconstruction algorithms implemented in the Virtual Instrument is given. The overview of the Virtual Instrument properties and performance are given in Section III, while its functionality is presented in Section IV. Concluding remarks are given in Section V.

II. THEORETICAL BACKGROUND

A. Compressive Sensing

Signal reconstruction based on CS principles requires some conditions to be satisfied [2]-[4]. If we observe a signal x of length N, it can be represented in terms of basis vectors ψ_i (of size $N\times 1$):

$$x = \sum_{i=1}^{N} s_i \psi_i = \psi s \ , \qquad (1)$$

where s_i is $N\times 1$ column vector of transform domain coefficients. The first condition required in CS is sparsity, i.e., the signal should be sparse in the domain ψ. Furthermore, we assume that only M measurements from x are available and arrange them in $M\times 1$ vector y. Selection of the samples has to be done in a random manner, to provide incoherent measurements, as the second CS requirement. Matrix that selects M out of N samples is called measurement matrix, and is denoted by Φ ($M\times N$):

$$y = \Phi x. \qquad (2)$$

Further, from (1) and (2), y can be written as:

$$y = \Phi x = \Phi \psi s = \theta s, \qquad (3)$$

where $\theta = \Phi\psi$ is called CS matrix ($M\times N$). The system of equations (3) is undetermined and can be solved using different optimization algorithms [5]-[10], [12]-[17].

B. Algorithms for 1D Signals Reconstruction

Some interesting optimization algorithms for 1D signals reconstruction will be described in the sequel. For instance, we focus to basis pursuit l_1-minimization method [5]-[7], [12], OMP [8]-[10], [12]-[13] algorithm and non-iterative threshold-based algorithm [14]-[17].

1. l_1-minimization using primal-dual interior point method: In order to recover signal x from its measurements $y = \Phi x$, the following optimization problem should be solved [2], [6]:

$$\min \left\| s^* \right\|_{l_1} \quad subject\ to \quad y = \theta s^* . \tag{4}$$

As l_1-norm is convex, linear programming can be used for solving this problem. The s^* is sparse vector obtained as the result of the optimization.

2. OMP algorithm: Using the CS matrix θ and vector y, OMP approximate signal s as linear combination of columns in θ. In each iteration, the set of columns is expanded with additional column that best correlates with the residual signal. The algorithm can be summarized as follows:

1. Set the approximation error $r_0 = y$, the initial solution to $s_0 = 0$ and $Z_0 = \varnothing$.
2. Do the following steps until the stopping criterion is met:

a) $Z_n = Z_{n-1} \cup \arg_i \max \left| \langle r_{n-1}, \theta_i \rangle \right|$,

b) $s_n = \arg\min_s \left\| r_{n-1} - Z_n s_{n-1} \right\|_2^2$, and $r_n = r_{n-1} - Z_n s_{n-1}$.

c) $n = n+1$ and $Z_n = Z_{n-1} \cup \arg_i \max \left| \langle r_{n-1}, \theta_i \rangle \right|$ until $n \le K$, where K is number of signal components.

3. Non-iterative signal reconstruction solution

The algorithm is based on the variances of estimation errors, calculated for each signal frequency [14]. The variance acts as a good indicator whether there is signal component on the observed frequency or not. Non-iterative algorithm provides fast reconstruction. It can be described as follows:

1. For a given number of measurements M on random positions q_M, calculate variance at each frequency:

$V(p) = \mathrm{var}(x(q_M) e^{-j2\pi p q_M / N})$, $p \in (1,N)$, N is signal length.

2. Find frequency positions p_{0_i} that satisfy relation:

$$p_{0_i} = \arg\min \{ V(p) < \alpha \cdot \max V(p) \}, \quad for \quad p = 1, ..., N ,$$

where α is $0.85 \le \alpha \le 0.95$.

3. Form CS matrix θ: matrix rows correspond to available measurements positions, while columns correspond to the frequencies p_{0i}.

4. Solve optimization problem: $\mathrm{X} = (\theta^* \theta)^{-1} \theta^* \mathrm{y}$

C. Algorithms for 2D Signals

The image is not strictly sparse in any transform domain, but its gradient can be observed as a sparse signal. Thus, instead of standard algorithms, the Total Variation (TV) [18]-[20] minimization is used for the reconstruction of the 2D signals. The TV of signal s represents the sum of the gradient magnitudes at each point (i, j):

$$\left\| s \right\|_{TV} = \sum_{i,j} \left| (Ds)_{ij} \right|, \tag{5}$$

where D represent operator described with relation:

$$D_{i,j} s = \begin{bmatrix} s(i+1, j) - s(i, j) \\ s(i, j+1) - s(i, j) \end{bmatrix}. \tag{6}$$

Minimization problem with TV could be described as:

$$\min_s TV\ (s) \quad subject\ to \quad y = \theta s . \tag{7}$$

It is important to note that all the implemented algorithms for 2D signals reconstruction are based on TV minimization procedure. The algorithms are shortly summarized below.

Algorithm 1 [19] takes samples from the 2D Fourier transform domain (2D DFT), along radial lines, and these samples will serve as measurements in the CS procedure.

Algorithm 2 [20] takes samples from 2D Discrete Cosine transform domain (2D DCT). Image is divided into blocks and the measurements are randomly taken from each block.

Algorithm 3 also uses 2D DCT as the observation domain. Small number of low-frequency DCT coefficients are used in this algorithm, while the rest of the measurement are randomly chosen pixels form middle and high frequency image regions.

III. VIRTUAL INSTRUMENT

The proposed Virtual Instrument is implemented in Matlab 7, and provides implementation of CS algorithms including the efficient analysis of reconstructed signals. It is important to emphasize that various signals can be uploaded or defined using signal parameters. Also, the performance of the algorithms could be compared by observing error between original and reconstructed signal, as well as execution time. The outlook of the instrument is shown in Fig. 1. It consists of two parts: part for 1D and part for 2D signals reconstruction. Both parts contain the **Definition** and the **Results** blocks. The **Definition** part contains: the **Signal choice** block, the **Percentage of measurements choice** block (for 1D part), the **Algorithm choice** block and the **Parameters choice** block (for 2D part). The part **Results** has **Numerical results** block and **Graphical results** block.

In the **Signal choice** block, the user chooses different audio signals (in 1D part), or may choose between medical and natural images (in 2D part). Additionally, for audio signals the option to hear sounds of original and reconstructed signal is implemented as well. Definition of the new signals (number of components, amplitudes and frequencies) is provided within **New signal** block - Fig. 1. Both, 1D and 2D parts have **Algorithm choice** block where one of the algorithms described in the Section II, can be selected and applied. **Numerical result** block contains errors between original and reconstructed signal (MAE - Mean Absolute Error, MSE - Mean Squared Error) for 1D signal, and PSNR for the 2D signals. Algorithm execution time is shown in this block as well. **Graphical result** block includes original and reconstructed signal plots in chosen domain, while **Graphical Error** represents graphic representation of absolute error between original and reconstructed signal (with zooming option).

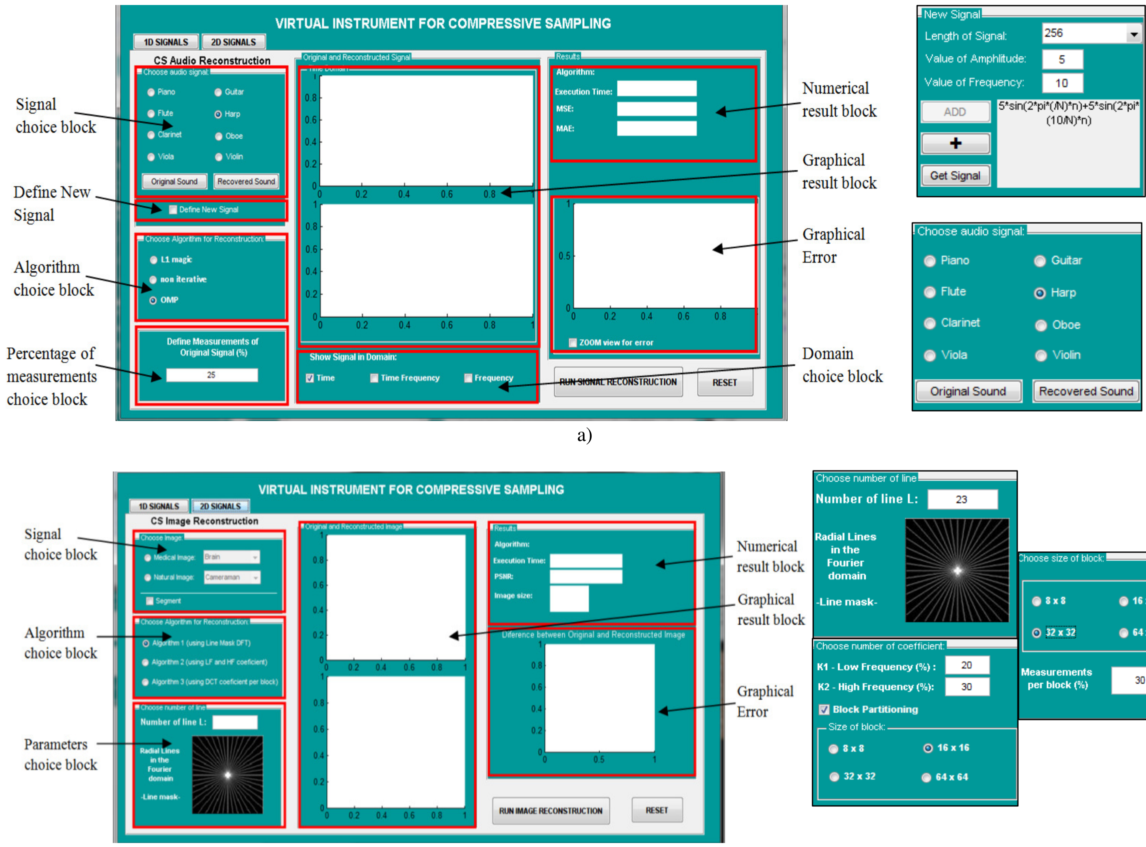

Fig.1. The outlook of the proposed Virtual Instrument (for 1D and 2D signals)

Graphical Error part in 2D signals contains image obtained as a difference between original and reconstructed image. **Percentage of measurements choice** block allows user to define number of measurements, while **Parameters choice block** has in-built parameters, depending on the 2D reconstruction algorithm. In **Domain choice block** for 1D signal, user can choose in which domain signal to be appeared: time, time-frequency and frequency domain.

By pressing Run Reconstruction button, the recovered 1D or 2D signal will appear in the corresponding window. For the comparison, output will be at the same panel as input signal. The numerical result (MAE, MSE, PSNR, execution time) will be shown and allow user to easily make conclusions about the performance of the algorithms.

IV. Functionality of Virtual Instrument

In this part the performances of the proposed Virtual Instrument are described on several signals. Firstly, 1D signals are observed.

A. 1D signals reconstruction

User can chose an audio signal representing a flute tone, for example. The observed signal is sparse in frequency domain and thus random samples are taken in time domain. In this example, the 40% of the randomly selected samples are taken for the reconstruction. Results obtained after running the l_1 reconstruction algorithm are shown in Fig. 2. Reconstruction quality for audio signal is measured by calculating MSE and MAE, as well as by plotting the MAE (Fig. 2). By comparing the sound of original and reconstructed signal, it has been confirmed that the quality of the reconstructed audio is preserved without introducing audible distortions. The l_1 reconstruction algorithm execution time for real audio signal is about 22 s. Note that the execution time depends on the length of the signal, as well as on the chosen reconstruction algorithm.

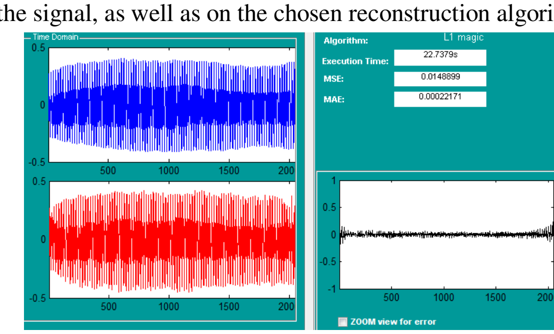

Fig. 2. Original (blue) and reconstructed (red) flute signal in time domain

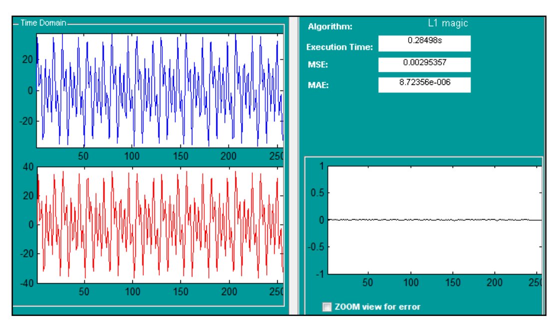

Fig. 3. Original and reconstructed user defined signal in time domain and reconstruction error

A Virtual Instrument also allows user to specify the signal with parameters such as number of harmonics, value of amplitude and value of frequency, and analyze performance of different algorithms for the defined signal (Fig. 3). The quality of the reconstruction will be better and error will be smaller by using larger number of measurements. Observing results, user can conclude how many measurement of original signal should be taken, in order to obtain good quality of reconstruction.

B. *CS Image Reconstruction (2D Signals)*

Virtual Instrument implements some of the commonly used algorithms for image reconstruction and allows 2D signals analysis as well. The results using algorithm based on 2D DCT random measurements are shown in this example. The medical image "Brain" is chosen for reconstruction and the 30% of the total number of samples are used in CS procedure. Original and reconstructed images are shown in the Fig. 4 (left column). On the same panel, execution time is presented, as well as the value of PSNR. Difference between original and reconstructed signal is shown as error image in the same figure (right column of the Fig. 4).

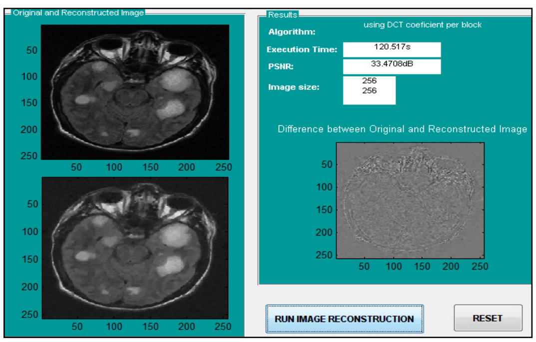

Fig. 4. Original and reconstructed image MRI Brain reconstructed by using Algorithm 3

V. CONCLUSION

The Virtual Instrument for CS signal reconstruction is proposed. It implements several algorithms for the reconstruction of 1D and 2D multimedia signals, such as audio signals and different types of images. The software is easy to use and provides a number of options for parameters settings and performance analysis. For instance, it has been shown that besides various CS algorithms, the users can change the number of measurements, as well as domain of sparsity. The proposed instrument provides the evaluation and comparison of different approaches (numerically and graphically). For all signal types, the algorithms execution time in sec is measured, as well. Therefore, the proposed Virtual Instrument could serve researches and practitioners in the CS field, and could be a very educative experimentation tool for those interested in CS for multimedia applications.

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