

# Compressive Sensing for Sparse Time-Frequency Representation of Nonstationary Signals in the Presence of Impulsive Noise

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## ABSTRACT

A modified robust two-dimensional compressive sensing algorithm for reconstruction of sparse time-frequency representation (TFR) is proposed. The ambiguity function domain is assumed to be the domain of observations. The two-dimensional Fourier bases are used to linearly relate the observations to the sparse TFR, in lieu of the Wigner distribution. We assume that a set of available samples in the ambiguity domain is heavily corrupted by an impulsive type of noise. Consequently, the problem of sparse TFR reconstruction cannot be tackled using standard compressive sensing optimization algorithms. We introduce a two-dimensional L-statistics based modification into the transform domain representation. It provides suitable initial conditions that will produce efficient convergence of the reconstruction algorithm. This approach applies sorting and weighting operations to discard an expected amount of samples corrupted by noise. The remaining samples serve as observations used in sparse reconstruction of the time-frequency signal representation. The efficiency of the proposed approach is demonstrated on numerical examples that comprise both cases of monocomponent and multicomponent signals.

**Keywords:** time-frequency analysis, compressive sensing, robust statistics, signal reconstruction, sparse representation

## 1. INTRODUCTION

Compressive sensing (CS) has been used in various applications with one-dimensional and two-dimensional signals<sup>[1]-[5]</sup>, as they appear in antenna arrays, indoor and SAR imaging, communications, remote sensing, biomedical and multimedia applications, etc. One of the main requirements imposed by the CS theory is signal sparsity. It means that the signals have concise representations when expressed in a proper basis. Namely, a signal can be accurately recovered if it is sparse in its own domain or in some of the transform domains such as DFT, DWT, DCT. In general, a signal which is  $K$  sparse in a specific domain can be completely characterized by  $M$  measurements ( $M > K$ ), although the total number of samples required by the Shannon-Nyquist theorem is  $N \gg M$ . The full signal reconstruction can be achieved through the convex optimization that uses sparsity as an important a priori information<sup>[6],[7]</sup>.

In most applications, the observed signals are typically sparse in one-domain and non-sparse in other. Observe that in the case of non-stationary signals, the time-frequency domain is used for signal representations<sup>[8]-[11]</sup>. Most of these signals are characterized by specific instantaneous frequency laws such as those corresponding to Doppler signatures of animate or inanimate targets<sup>[12]-[14]</sup>. The power localization property in joint-domain representations renders these signals sparse not only in the time-frequency, but also in the ambiguity domain. However, attenuating or removing the cross-terms necessitates discarding ambiguity domain points that are far from the origin. The selection of few observations in the ambiguity domain near the origin enables the formulation of under-determined linear model. In this case, sparse signal reconstruction techniques would provide high resolution time-frequency signal representation, thereby allowing accurate IF estimations<sup>[15],[16]</sup>.

In this paper, we deal with ambiguity domain observations affected by significant amount of impulsive noise. Here, it is important to emphasize that the problem formulation is different from the concept introduced in previous paper<sup>[16]</sup>. Therein, the noisy signal in time domain is processed using robust statistics, assuring that afterwards the ambiguity function is noise free. In other words, the measurements taken from ambiguity function are not affected by noisy pulses. The underlying problem is accurate reconstruction of sparse time-frequency representation when impulse noise is

encountered in the ambiguity domain (as domain of observations). The standard CS reconstruction techniques<sup>[15],[16]</sup> will not be able to provide desirable results. Therefore, we propose a robust method to map the noisy ambiguity domain observations to the non-noisy sparse time-frequency representation. The measurements around the origin in the ambiguity domain are used in  $\ell_1$  minimization to yield the sparse cross-term free TFR. In order to deal with impulse noise, we apply robust estimation approach directly to the ambiguity domain rather than the time-domain, which has been usually the case. One efficient robust approach is based on the concept of L-estimation and  $\alpha$ -trimmed filter form<sup>[17]</sup>. It can be suitable not only in the presence of impulse noise, but also in the presence of mixed Gaussian and impulse noise. We show that the proposed approach significantly eliminates the influence of noisy pulses, providing better time-frequency representation compared to the standard approach. Simulation examples include noisy signals (both monocomponent and multicomponent) with different instantaneous frequency laws.

The paper is organized as follows. The ambiguity domain and time-frequency domain representations have been reviewed in Section II. The compressed sensing in the time-frequency analysis is given in Section III, including the proposed modification for the case of impulse noise. The simulation results are illustrated in Section IV, while the concluding remarks are given in Section V.

## 2. AMBIGUITY FUNCTION AND TIME-FREQUENCY SIGNAL ANALYSIS

Time-frequency analysis has been widely used to deal with signals characterized by time-varying spectral content<sup>[18]-[25]</sup>. Very often it has been employed to estimate the signal's instantaneous frequency law, bringing important information about the physical processes in real applications, such as communications, speech and audio analysis, radars<sup>[18]-[20]</sup>, biomedicine<sup>[21]</sup>, multimedia<sup>[23]-[25]</sup>. One of the commonly used time-frequency distributions is the Wigner distribution defined by:

$$WD(t, \omega) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{2})x^*(t - \frac{\tau}{2})e^{-j\omega\tau} d\tau. \quad (1)$$

The Wigner distribution provides an ideal concentration for linear frequency modulated signals. However for signals with nonlinear instantaneous frequency, it produces inner-interferences caused by the third and higher order phase derivatives. The more serious drawback appears in the case of multicomponent signals, when the Wigner distribution produces undesired components called cross-terms. They reside between signal's auto-terms, at the position of their arithmetic mean, making the Wigner distribution ineffective in most of the practical applications. In order to deal with cross-terms, ambiguity domain based time-frequency distributions are widely studied<sup>[26]-[29]</sup>. The ambiguity function is defined as,

$$A(\tau, \theta) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{2})x^*(t - \frac{\tau}{2})e^{-j\theta t} dt. \quad (2)$$

The ambiguity function is a counterpart of the Wigner distribution as they are related by the two-dimensional Fourier transform,

$$WD(t, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\tau, \theta)e^{j(\theta t - \omega\tau)} d\tau d\theta.$$

The main advantage of using the ambiguity domain is based on the fact that the auto-terms are located at and near the origin. Thus, the cross-terms can be suppressed, or significantly attenuated, by using low-pass filtering function realized by using different types of kernel  $c(\tau, \theta)$ ,

$$A_f(\tau, \theta) = A(\tau, \theta)c(\tau, \theta). \quad (3)$$

The class of time frequency distributions based on the filtered ambiguity function is obtained as:

$$CD(t, \omega) = \mathbb{F}_{2D}^{-1}\{A(\tau, \theta)c(\tau, \theta)\}, \quad (4)$$

and it is referred to as the Cohen class of distributions. A number of distributions belonging to this class have been introduced to deal with multicomponent signals, whereas they differ only by the kernel shape. However, it is important to note that reduced interference kernels negatively influence the auto-terms concentration. In other words, there is a trade-off between cross-terms reduction and auto-terms concentration<sup>[28]</sup>.

### 3. COMPRESSIVE SENSING IN TIME-FREQUENCY ANALYSIS

#### 3.1 The compressive sensing concept

Compressive sensing is generally related to the process of simultaneous sensing and compression of signals that relies on linear dimensionality reduction. The signal of interest  $x$  is not sampled according to the Shanon-Nyquist sampling theorem that requires a certain number of  $N$  samples. Instead, we acquire an arbitrary set of  $M$  linear measurements, where  $M \ll N$ . This linear set of measurements can be acquired using a certain CS matrix  $\Phi$  of size  $M \times N$ . The corresponding incomplete set of samples is called the measurement vector and can be defined as:

$$y = \Phi x . \quad (5)$$

The CS matrix  $\Phi$  includes independent projections that allow the recovery of full data set,  $N$ , using just few observations,  $M$ . Namely, since  $M < N$  holds, the sensing matrix  $\Phi$  is rank-deficient, which means that for a certain signal  $x_p \in \mathbb{R}^N$ , an infinite number of signals  $x_i$  might have the same measurements:

$$y_p = \Phi x_p = \Phi x_i .$$

Hence, the CS matrix should be defined such that the measurements  $y_i = \Phi x_i$ ,  $y_j = \Phi x_j$  provide unique identification of different signals  $x_i$  and  $x_j$ . One of the major requirements that should be satisfied in CS is signal sparsity, meaning that a signal has concise representation in its own or a certain transform domain. Such a signal can be well-approximated by using a small number of non-zero coefficients, if a suitable basis is used.

Consider the signal representation in a certain basis  $\{\psi_i\}_{i=1}^N$  for  $\mathbb{R}^N$ . The signal  $x \in \mathbb{R}^N$  can be represented as:

$$x = \sum_{i=1}^N \psi_i \vartheta_i .$$

If the basis vectors  $\psi_i$  are columns of the matrix  $\Psi$  of size  $N \times N$ , and the transform coefficients  $\vartheta_i$  are written in the vector form  $\vartheta$ , then the signal can be expressed as follows:

$$x = \Psi \vartheta . \quad (6)$$

If the number of non-zero coefficients in  $\vartheta$  is  $K \ll N$ , then we may say that  $x$  is  $K$ -sparse in domain  $\Psi$ . Taking  $M$  linear measurements of the signal  $x$ , using the CS matrix, we obtain,

$$y = \Phi x = \Psi \Phi \vartheta . \quad (7)$$

An additional constraint, which ensures accurate signal reconstruction from its measurements, is that the sensing matrix  $\Phi$  and the fixed basis matrix  $\Psi$  represent a low coherence pair. Thus, the matrix  $\Phi$  is usually a random matrix, since it exhibits a very low coherence with a basis matrix.

### 3.2 CS problem formulation in the ambiguity domain

Improved time-frequency signal power localizations can be achieved by using the compressed sensing approach and exploiting sparsity in the time-frequency domain<sup>[15]</sup>. For most of the signals appearing in real applications, the time-frequency representation contains a small number of non-zero values. The sparsity assumption is defined as follows: the  $N \times N$  time-frequency representation of signal with  $K$  components (where  $K \ll N$ ) should have at most  $K \cdot N$  non-zero points. Due to the ability to reduce cross-terms by focusing on the region near the origin in the ambiguity-domain, the latter becomes the preferred observation-domain, as discussed earlier. We, therefore, formulate the CS problem accordingly. In that sense, the observation vector is obtained as a set of measurements from the ambiguity domain:

$$y = \{A(\tau, \theta) | (\tau, \theta) \in \Omega\}, \quad (8)$$

where, in order to avoid the cross-terms, the measurements are taken from specific ambiguity region  $\Omega$  around the origin. Therefore, in the CS problem, the observations selection region should provide similar low-pass performance as the kernel function. Furthermore, the transform domain matrix is obtained as a two-dimensional Fourier transform,

$$\Psi = \mathbb{F}_{2D}, \quad (9)$$

which, together with the random CS sensing matrix  $\Phi$ , constitutes a low-coherence pair. The combined matrix is referred to as the representation dictionary and will be denoted as  $\Upsilon = \Phi\Psi$ . Since the measurements are taken from the predefined ambiguity domain region,  $\Omega$ , the matrix  $\Upsilon$  is actually obtained as,

$$\Upsilon = \Phi\Psi = \mathbb{F}_{2D}(\Omega). \quad (10)$$

Here, it is important to emphasize that a suitable set of ambiguity domain samples  $\Omega$  can be obtained by applying an appropriate ambiguity domain mask, formed as a small area around the origin. The resulting transform domain vector consists of the coefficients from the sparse time-frequency representation:

$$\vartheta = \{WD(t, \omega) | t \in (t_1, t_N), \omega \in (\omega_1, \omega_N)\}. \quad (11)$$

Due to the sparsity in the time-frequency domain, most of the coefficient in  $\vartheta$  should be zero-valued.

### 3.3 Robust statistics tailored to CS reconstruction in ambiguity domain

The CS sparse solution should be obtained by solving the system defined by (7), where the optimization system variables are defined by (8), (10) and (11). This system is under-determined and can have infinitely many solutions. According to the theory, the localized distribution with the smallest possible number of non-zero coefficients can be obtained as a solution of  $\ell_0$ -norm minimization of the time-frequency distribution. However, in practice we may use the near-optimal solutions based on the  $\ell_1$ -norm minimization:

$$\min \|\tilde{\vartheta}\|_{\ell_1} \quad \text{subject to } y = \Upsilon \tilde{\vartheta}. \quad (12)$$

This problem is known as Basis Pursuit, and it can be solved by commonly used simplex and interior point methods (e.g., primal-dual interior point method). Some of the commonly used algorithms for sparse signal reconstruction are also the Orthogonal matching pursuit and Block orthogonal matching pursuit algorithm. However, the problem appears in the presence of noise, especially when the underlying ambiguity measurements are corrupted by impulse kind of noise. Namely, the linear measurements are severely degraded, with original information masked by large noise amplitudes spread across the measurements. The corrupted samples will cause standard reconstruction algorithms to fail in their attempts to recover an accurate sparse time-frequency representation. One solution could be to define a robust measurement procedure which is not based on linear projections in order to avoid the impulse noise. In our case, we

have a predefined measurements defined by the ambiguity domain mask around the origin. Thus, we propose a solution which includes robust statistics into CS reconstruction technique in the ambiguity domain. Namely, we seek to provide a noise free version of the initial transform domain vector  $\vartheta_0$  or, in other words, robust initial transformation to the time-frequency domain. This is achieved using the L-statistics approach. The L-estimators are defined as linear combinations of order statistics, and can be used even for the mixed noise type. The L-statistics approach involves sorting out data samples taking care of the corresponding complex frequencies (in the exponents), and then removing the highest values. The minimization problem can be finally formulated as follows:

$$TFR = \arg \min_{\vartheta} \|\vartheta\|_1, \quad Y\vartheta - y = 0 \Big|_{(\tau, \theta) \in \Omega}, \quad (13)$$

$$\text{with initial transform: } \vartheta_0 = \Re\{A(\tau, \theta)\} = \sum_{i=1}^{M(1-2\alpha)+4\alpha} AL_i(\tau, \theta), \quad (14)$$

$$AL(\tau, \theta) = \text{sort}\{A(\tau, \theta)e^{-j2\pi\tau k/M} e^{-j2\pi\theta l/M}\},$$

where  $(\tau, \theta) \in \Omega$  and  $\text{card}\{\Omega\} = M$

Note that the sorting operation is performed in non-decreasing order. In order to provide noise free  $x_0$ , we omit  $2\alpha(M-2)$  of the highest elements in  $AL(\tau, \theta)$ , while the mean is calculated over the rest of the values. Therefore, the proposed modified approach can be observed as the L-statistics based  $\ell_1$ -norm minimization.

#### 4. SIMULATION RESULTS

In order to illustrate the advantages of the proposed method, let us observe a set of noisy measurements in the ambiguity domain. The monocomponent sine frequency modulated signal is considered:

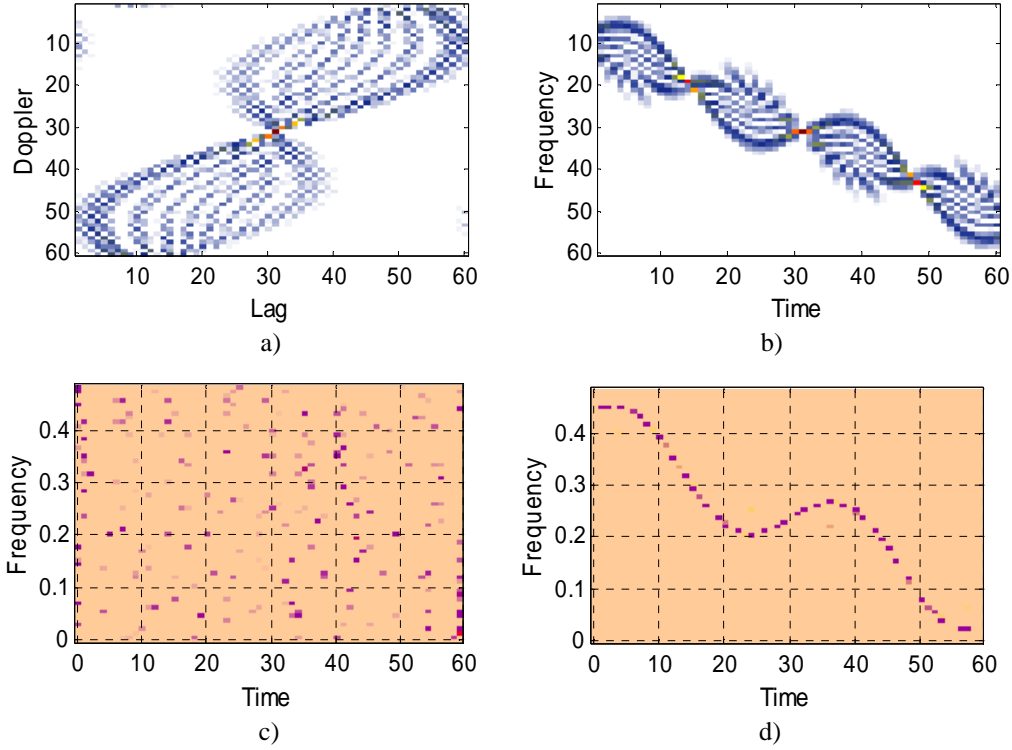
$$x(t) = e^{j(-5\sin(1.8\pi t - \pi/2) + 10\pi t^2)}.$$

The ambiguity function is calculated for the considered signals and the ambiguity domain measurements are taken from the predefined ambiguity domain region:  $\Omega \in \{\tau_0 - \beta N_1 : \tau_0 + \beta N_1, \theta_0 - \beta N_2 : \theta_0 + \beta N_2\}$ , where  $(\tau_0, \theta_0)$  is the coordinate of the central point in the ambiguity domain. The parameter  $\beta$  is set to 0.15, and it is actually defined such that 30% of samples along each coordinate are included in the ambiguity mask. The total size of ambiguity function is  $N_1 \times N_2$ , and in the examples  $N_1 = N_2 = 60$  is used. The assumption is that 10% of the measurements are corrupted by strong noisy peaks. Consequently, the direct CS based sparse time-frequency reconstruction cannot provide satisfactory results. In order to remove noisy peaks, we need to discard sufficient amount of samples using L-statistics, to ensure elimination of most of the noisy pulses. In most cases, we can discard more samples than those corrupted by the noise. Hence, we observe the case when the parameter  $\alpha$  (which defines the number of discarded samples) is chosen as:

$$M(1-2\alpha) + 4\alpha = 0.85\% M$$

In essence, we discard 15% of the highest values in the initial transform domain vector  $\vartheta_0$  in (14).

The non-noisy ambiguity function (corresponding to full data set) and the corresponding standard Wigner distribution of monocomponent signal  $x(t)$  are shown in Figure 1.a and b, respectively. Note that, due to the phase non-stationarity, we need to deal even with the strong inner-interferences that appear in the Wigner distribution. The results of standard CS reconstruction from noisy measurements are given in Figure 1.c, showing that it is not suitable in the presence of impulse noise. The sparse time-frequency representation, obtained using the proposed L-statistics based  $\ell_1$ -norm minimization are given in Figure 1.d.

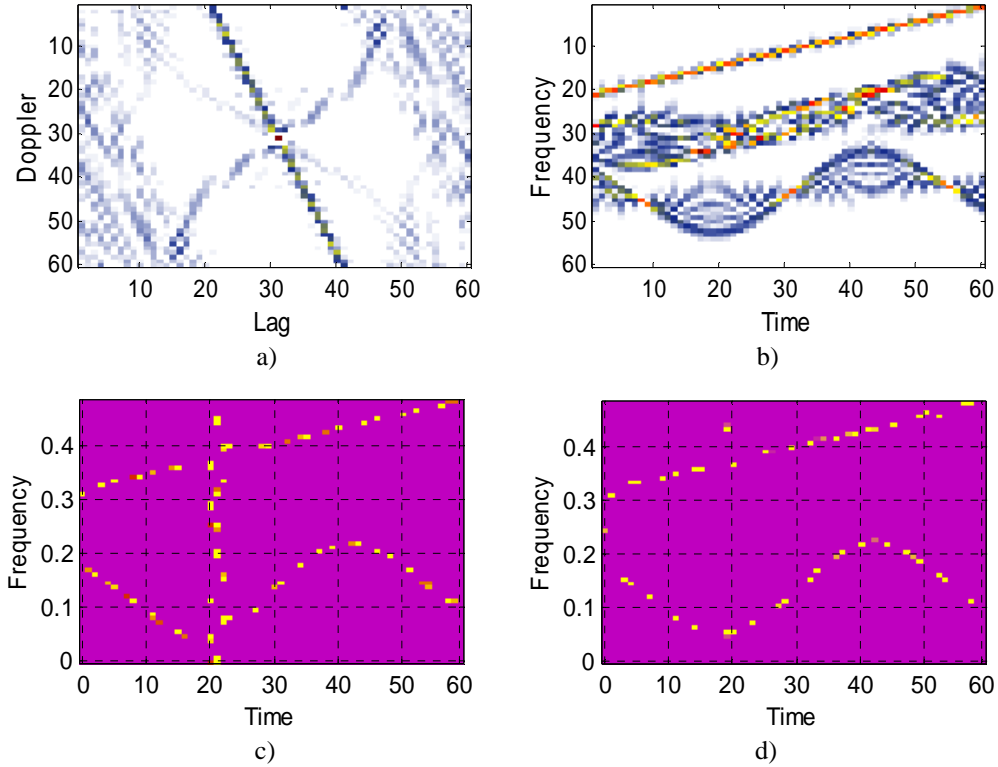


**Fig 1.** Signal  $x(t)$ : a) Standard ambiguity domain representation of original signal; b) standard Wigner distribution of original signal; c) Sparse time-frequency reconstruction results from the set of noisy measurements in the ambiguity domain, b) sparse time-frequency reconstruction results obtained using the proposed method

A more challenging situation arises when dealing with a non-stationary multicomponent signal, where, in addition to noise, strong cross-terms between each component pairs can obscure the individual component power distribution. Consider the signal that consists of a chirp and a cosine frequency modulated component:

$$y(t) = e^{j(16/5\cos(3/2\pi t) + 6\cos(\pi t) + 12\pi t)} + e^{-j(5\pi t^2 + 20\pi t)}.$$

The non-noisy ambiguity function (corresponding to full data set) and the Wigner distribution of considered multicomponent signal are shown in Figure 2.a and b, respectively. The cross-terms are evident. In this example, we assume that the noise is concentrated as a burst within the narrow time range in the time-frequency domain. The results of standard  $\ell_1$ -based reconstruction of the noisy data are shown in Figure 2.c, where the cross-terms are eliminated, but due to the nature of the additive noise, there is a persistent burst component. The proposed L-statistics based  $\ell_1$  minimization again provides both cross-terms noise free results, as shown in Figure 2.d.



**Fig 2.** Signal  $x(t)$ : a) Standard ambiguity domain representation of original signal; b) standard Wigner distribution of original signal; c) Sparse time-frequency reconstruction from the set of noisy measurements in the ambiguity domain, b) sparse time-frequency reconstruction results obtained using the proposed method

## 5. CONCLUSION

This paper dealt with CS and time-frequency signal representation (TFSR). The considered nonstationary signals were sparse in the time-frequency domain. The CS algorithm was applied to the ambiguity domain observations to provide high resolution time-frequency representation suitable for instantaneous frequency estimation. We focused on the case where the observations are corrupted by impulse noise. We used a two-dimensional L-statistics to sort out the ambiguity function samples and select those of high signal to noise ratio. The L-statistics provided a noise free sparse TFR. The efficiency of the proposed approach was demonstrated by numerical examples that comprised both cases of monocomponent and multicomponent signals.

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