Gradient Algorithm Based ISAR Image Reconstruction From the Incomplete Dataset

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***Abstract*—In the inverse synthetic aperture radar (ISAR) number of target reflectors is small resulting in the fact that ISAR images are sparse. Since the ISAR image is obtained using the Fourier transform of the input signal then this problems can be treated within processing of sparse signals. The compressive sensing (CS) theory proves that, under some conditions, exact reconstruction of sparse signals is possible based on the reduced set of observations. Here we will present a gradient based reconstruction algorithm and apply it to the several ISAR setups. In contrast to the common reconstruction algorithms where the signal in its sparsity domain is reconstructed, this algorithm solves minimization in an indirect way, by calculating the missing samples/measurements. Obtained results show that the presented simple reconstruction algorithm is reliable and robust. Common problems in ISAR imaging, like uncompensated motion or target nonuniform motion, make ISAR image only approximately sparse. The presented procedure can provide useful results in these cases as well.**

***Index Terms*—ISAR, Compressive sensing, Gradient algorithm**

I. Introduction

The Inverse Synthetic Aperture Radar (ISAR) is a radar system capable to provide image of the illuminated moving target [1], [2]. ISAR images are obtained by target motion compensation in a such way that the rotational motion only remains in the received signal. Two-dimensional Fourier transform can be applied to the received and preprocessed signal to obtain a high resolution target image.

In Compressive Sensing (CS) the main focus is on extracting required information from a reduced set of observations [3], [4]. The CS can be applied in signal processing to the signal analysis and reconstruction using a reduced number of samples [5]. Although first application of the CS theory was in computed tomography today it is spread over wide area of practical applications including, but not limited to, biomedical signal analysis [6] and sparse signals reconstruction [5], [7]. Application of the CS theory to radar signal processing is important research topic and many results in this area are published recently [8]–[16].

In order to explain possible benefits of applying CS to ISAR we can consider two scenarios. In the first scenario assume that radar signal is jammed so that some of the radar pulses could be marked as jammed. If we cannot eliminate jammer then it is better to avoid such pulses in the analysis. In this case some radar pulses are intentionally omitted from the further processing. They could be reconstructed based on the available pulses by using the CS reconstruction techniques.

In the second scenario we can assume that radar does not transmit every pulse. Some pulses are intentionally omitted, for example, to make our system energy more efficient or to lower possible electromagnetic interference with neighboring devices. Although the reasons for pulses being missing are different, the reconstruction process is the same in all scenarios.

Here we will briefly review the ISAR signal model and a simple gradient algorithm for missing samples reconstruction for complex valued signals. The algorithm is then applied to the simulated ISAR cases and results are presented for various percentage of available pulses.

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II. ISAR signal model

Consider a continuous wave (CW) radar that transmits a signal in the form of a coherent series of linearly frequency modulated chirps [1]:

$$v_p(t) = \begin{cases} e^{j\pi B f_r t^2} & \text{for } 0 \le t \le T_r \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where T_r is the repetition time, $f_r = 1/T_r$ is the repetition frequency, and B is the waveform bandwidth. The transmitted signal consists of N such chirps:

$$v(t) = e^{-j\omega_0 t} \sum_{n=0}^{N-1} v_p(t - nT_r), \tag{2}$$

where ω_0 is the radar operating frequency. The total signal duration is $T_c = NT_r$ and represents the CIT (coherent integration time).

Without loss of generality consider a single point target scenario. If the target distance from the radar is d, then the received signal is delayed with respect to the transmitted signal (2) by $t_d = 2d/c$, where c is the speed of light. Phase of the received signal is changed as $\phi = 2kd = 4\pi d/\lambda = 4\pi d f_0/c = 2\omega_0 d/c$. Thus, the received signal is of the form

$$u(t) = \sigma e^{-j\omega_0 (t-2d/c)} \sum_{n=0}^{N-1} v_p(t - 2d/c - nT_r), \tag{3}$$

where σ is the reflection coefficient. Received signal (3) is mixed (multiplied) with appropriately delayed and conjugated transmitted signal (2) and the result is low pass filtered. The radar output is of the form

$$q(n,t) = \sigma e^{j\omega_0 2d/c} e^{-j2\pi B f_r (t-nT_r)2d/c}. \quad (4)$$

Motion compensation is performed in a such way that only the rotational motion remains in the received signal [1], [2], [17]. The signal is sampled in time with $t - nT_r = mT_s$ and the output signal is obtained as a two-dimensional discrete sinusoid of the form

$$x(m,n) = \sigma e^{j\omega_r m} e^{j\omega_{cr} n}, \quad (5)$$

where the considered point range coordinate is proportional to ω_r and the cross-range coordinate is proportional to ω_{cr}.

ISAR image is obtained as a two-dimensional (2D) Fourier transform of the received signal

$$X(m',n') = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(m,n) e^{-j\frac{2\pi}{M}(mm'+nn')}. \quad (6)$$

In the considered case $X(m',n')$ reduces to a single non-zero value located at the positions proportional to ω_r and ω_{cr}. For a target with K_R reflectors, the sampled radar output is a sum of components of form (5) and corresponding ISAR image has K_R non-zero points. This mean that signal $x(m,n)$ is sparse signal with sparsity equal to the total number of target points.

If we consider a single range bin m_0, the corresponding signal $x(n) = x(m,n)|_{m=m_0}$ is also sparse one-dimensional signal with sparsity equal to the number of target points within the considered range bin. This property of ISAR signal will be used to reconstruct the range bins, i.e., the whole ISAR image, based on the reduced set of samples.

III. ISAR Image Reconstruction

Since we know that the ISAR signal is sparse we can apply CS reconstruction methods in order to obtain full image from a reduced set of observations. In CS it is shown that the number of observations should be at least twice larger than signal sparsity. In the ISAR case this condition is satisfied even for a small number of observations since the expected sparsity (target points in a single range) is small.

We will use the reconstruction algorithm described in [18] suitable for complex-valued signals. It is based on the algorithm for real-valued signal reconstruction proposed in [7], [19]. Primary goal during the reconstruction process of common reconstruction algorithms is to find the signal in the domain of sparsity. In this case it is the Fourier transform $X(k)$ of the signal $x(n)$, using the available samples. Vector notation of $X(k)$ is $\mathbf{X}$. Assume that the signal $x(n)$ is sparse in the DFT domain and that some pulses are unavailable. Denote unavailable (missing) pulse positions with $n \in \mathbb{N}_Q = \{q_1, q_2, ..., q_Q\}$, $y(i) = x(q_i)$. The task is to minimize $\|\mathbf{X}\|_1$ subject to the available signal values $y(i) = x(q_i)$, $i = Q+1, Q+2, ..., N$. Compared to the other CS reconstruction algorithms, the presented algorithm is specific in the sense that it does not solve the problem by a direct finding of the signal values $X(k)$ in the sparsity domain. This algorithm calculates the missing sample (measurement) values in order to find a complete set of samples $x(n)$ which minimizes $\|\mathbf{X}\|_1$. This kind of approach, where the missing samples/measurements are the reconstruction variables, is possible when the complete set of samples/measurements exists, like in the case of a signal and its common transformation domains. Here we will review the reconstruction procedure.

Consider a single range bin. Note that it is possible that the considered signal does not contain any target point at all (no target points within the considered range). In this case we should not proceed to further analysis. We can set whole range bin signal to zero. This case can be detected by introducing threshold based on the signal energy contained in each range bin. In the examples we use a threshold between 1% and 5% of the maximal energy detected in a single range bin.

The basic idea is to start from the minimum energy solution $y^{(0)}(n)$. It is equal to the available samples of the considered signal with zero-valued missing samples. Now we should vary real and imaginary part of the missing sample values for $\pm\Delta$ where Δ is appropriately chosen variation step. We should analyze influence of these variations to the concentration measure [20] of the signal in frequency domain. The measures, defined as a sum of absolute values, are used in order to obtain an estimation of the measure gradient. Next, the missing sample values are adjusted and the whole procedure is repeated. Good starting choice of the algorithm parameter Δ is signal magnitude $\Delta = \max|x(n)|$.

The iterative procedure of the reconstruction algorithm can be summarized as

Step 1: For each missing sample at $n = q_i$ we form four signals $y_1(n)$, $y_2(n)$, $y_3(n)$, and $y_4(n)$ in each next iteration as:

$$\begin{aligned}
y_1^{(k)}(n) &= \begin{cases} y^{(k)}(n) + \Delta & \text{for } n = q_i \\ y^{(k)}(n) & \text{for } n \neq q_i \end{cases} \\
y_2^{(k)}(n) &= \begin{cases} y^{(k)}(n) - \Delta & \text{for } n = q_i \\ y^{(k)}(n) & \text{for } n \neq q_i \end{cases} \\
y_3^{(k)}(n) &= \begin{cases} y^{(k)}(n) + j\Delta & \text{for } n = q_i \\ y^{(k)}(n) & \text{for } n \neq q_i \end{cases} \\
y_4^{(k)}(n) &= \begin{cases} y^{(k)}(n) - j\Delta & \text{for } n = q_i \\ y^{(k)}(n) & \text{for } n \neq q_i \end{cases},
\end{aligned} \quad (7)$$

where k is the iteration number. Constant Δ is used to determine whether the real and imaginary parts of the considered signal sample should be decreased or increased.

Step 2: Estimate the differences of the signal transform measure (norm-one measure in our examples) as

$$g_r(q_i) = \mathcal{M}\left[\mathrm{DFT}[y_1^{(k)}(n)]\right] - \mathcal{M}\left[\mathrm{DFT}[y_2^{(k)}(n)]\right] \quad (8)$$

$$g_i(q_i) = \mathcal{M}\left[\mathrm{DFT}[y_3^{(k)}(n)]\right] - \mathcal{M}\left[\mathrm{DFT}[y_4^{(k)}(n)]\right] \quad (9)$$

where $\mathcal{M}[\cdot]$ is norm 1 measure (sum of absolute values of the considered DFTs).

Step 3: Form a gradient vector $\mathbf{G}^{(k)}$ with the same length as the signal $x(n)$. At the positions of the available samples, this vector has value $G^{(k)}(n) = 0$. At the positions of missing samples gradient vector values are

$$G^{(k)}(q_i) = g_r(q_i) + j\, g_i(q_i), \tag{10}$$

calculated by (8) and (9).

Step 4: Correct the values of $y(n)$ iteratively by

$$y^{(k+1)}(n) = y^{(k)}(n) - \frac{1}{N} G^{(k)}(n). \tag{11}$$

Repeating the presented iterative procedure, the missing values will converge to the true signal values, producing the minimal concentration measure in the transformation domain. Stopping criterion and criterion for reduction algorithm parameter Δ are discussed in [18]. The parameter Δ is reduced by $\sqrt{10}$ when the angle between successive gradient vectors is close to $180°$ (for example higher than $170°$). The iterative procedure is stopped when there is no significant change in $y(n)$ for two successive values of Δ.

In theory, the reconstructed signal uniqueness can be checked using the restricted isometry property with appropriate constants. However its calculation is an NP hard problem. Uniqueness of the obtained solution can be checked in an efficient way using the recently proposed uniqueness theorem [21]. After the signal $x(n)$ is reconstructed and its sparse transform $X(k)$ is obtained, with nonzero values at $k_i \in \mathbb{K}_s = \{k_{01}, k_{02}, ..., k_{0s}\}$, this theorem checks if there is another signal $X(k)+Z(k)$ with the same or lower sparsity. Here $Z(k)$ is the DFT of $z(n)$, where $z(n)$ assume arbitrary values at the missing sample positions $n \in \mathbb{N}_Q$ and $z(n) = 0$ for $n \notin \mathbb{N}_Q$. Note that the available sample values and positions in $x(n)$ and the nonzero positions in reconstructed $X(k)$ are fixed, while the positions of zero and nonzero values in $Z(k)$ could change to produce minimal possible sparsity of $X(k) + Z(k)$.

Theorem [21]: Consider reconstruction of the signal $x(n)$ that is sparse in the DFT domain with unknown sparsity. Assume that the signal length is $N = 2^r$ samples and that Q samples are missing at the positions $n \in \mathbb{N}_Q = \{q_1, q_2, ..., q_Q\}$. Also assume that the reconstruction is performed and that the DFT of reconstructed signal is of sparsity s. Assume that the positions of the reconstructed nonzero values in the DFT are $k_{0i} \in \mathbb{K}_s = \{k_{01}, k_{02}, ..., k_{0s}\}$ Reconstruction result is unique if the inequality

$$s < N - \max_{h=0,1,...,r-1} \left\{2^h \left(Q_{2^h} - 1\right) - s + 2S_{2^{r-h}}\right\}$$

holds. Integers Q_{2^h} and $S_{2^{r-h}}$ are calculated as

$$Q_{2^h} = \max_b \{\mathrm{card}\{q : q \in \mathbb{N}_Q \text{ and } \mathrm{mod}(q, 2^h) = b\}\}$$
$$b = 0, 1, ..., 2^h - 1$$
$$S_{2^{r-h}} = \sum_{l=1}^{Q_{2^h}-1} P_h(l)$$
$$P_h(l) = \mathrm{sort}_p\{\mathrm{card}\{k : k \in \mathbb{K}_s \text{ and } \mathrm{mod}(k, 2^{r-h}) = p\}\}$$
$$p = 0, 1, ..., 2^{r-h} - 1$$

where $P_h(1) \le P_h(2) \le ... \le P_h(2^{r-h})$ and $\mathrm{card}\{\cdot\}$ is cardinal number (number of elements) of the considered set.

IV. Reconstruction examples

Presented procedure is demonstrated on three examples. Signal analyzed in the first example is simulated radar output signal that is sparse in each range bin with no more than 7 target points within the considered range. This signal is well suited to the presented procedure. The signal is consisted of 256 pulses with 64 samples within each pulse. We assume that some, randomly positioned pulses are missing (or omitted) and perform ISAR image reconstruction with complex gradient based reconstruction algorithm. Results are presented in Fig. 1 for 30%, 50%, and 70% of available pulses. The ISAR images obtained directly from the available pulses (with samples all samples corresponding to missing pulses set to zero, minimum energy condition) are shown on the left side. Corresponding images obtained after the reconstruction are given on the right. It is clear that the presented algorithm is able to reconstruct the original image in all considered cases.

More realistic scenario is used in the next examples. Simulated Boeing 727 and Mig 25 signals are used. These signals are commonly used as test signals for various ISAR improvement methods. Mig 25 signal has 512 pulses with 64 samples within each pulse. Number of pulses in Boeing 727 signal is 256. These signals are only approximately sparse in each range bin. Here we do not expect exact reconstruction from the incomplete set of data. Moreover, ISAR images are blurred due to possible nonuniform or uncompensated target motion.

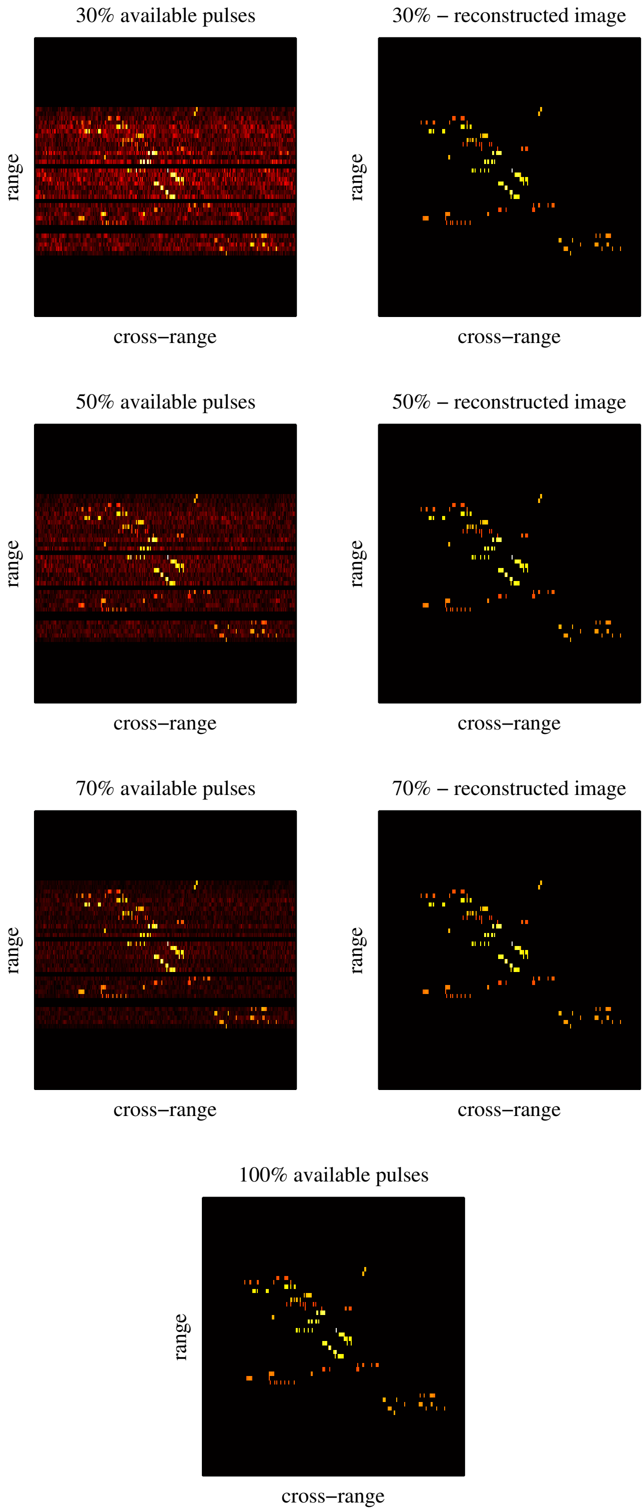
The results are presented in Fig. 2 and Fig 3. In the case of 70% available samples (third row in the figures) the reconstructed image is almost equal to the original one. Small differences are notable in the case when half of the pulses are unavailable (second row in the figures). Reconstructed image in the case with 30% available pulses produce ISAR image that is better than the image obtained without reconstruction (presented left), but in this case difference with ISAR image obtained by using full dataset (the last subplot) is notable.

From the presented examples we can see that reconstruction of missing pulses is possible and that the reconstruction procedure is very simple and robust to the signal sparsity assumption.

Calculation complexity of the proposed algorithm is $O(N_{it} N \log_2 N)$ where M is number of samples within one range bin and N_{it} is total number of iterations in the algorithm.

V. Conclusion

Common ISAR signals are sparse in the Fourier transform domain with a low number of non-zero values in this domain. Such signals are well suited to the Compressive Sensing reconstruction techniques. In this paper, based on the model of ISAR signal, we present an efficient and robust algorithm for the ISAR image reconstruction using a reduced set of observations. The proposed method is tested on simulated examples. The obtained results are satisfactory even in realistic



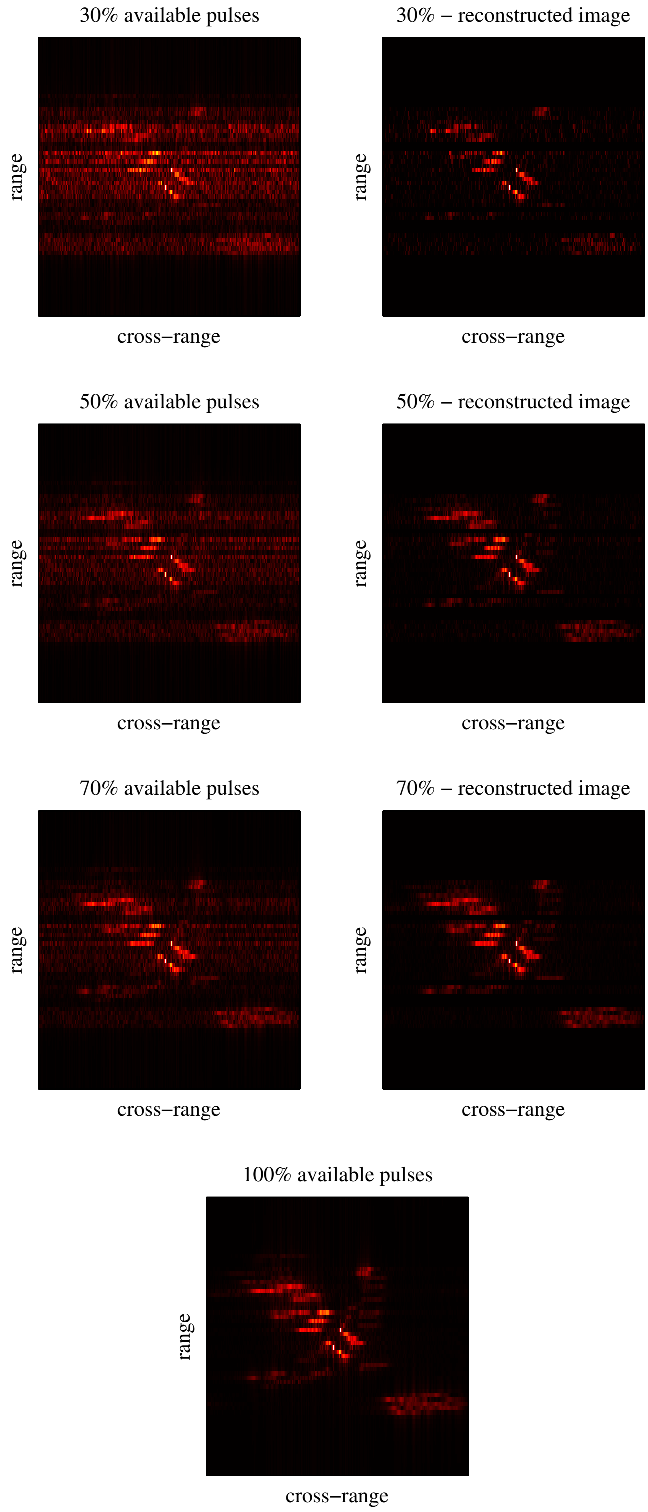


Fig. 1. Simulated example with ideally sparse signal. Maximal sparsity in each range bin is 10. The reconstruction is performed based on 30%, 50%, and 70% of the available pulses and reconstructed images are shown on the right side. Images without missing pulses reconstruction are shown on the left. In the last row ISAR image with full dataset is presented.

Fig. 2. Boeing 727 example. This signal is only approximately sparse. The reconstruction is performed based on 30%, 50%, and 70% of the available pulses and reconstructed images are shown on the right side. Images without missing pulses reconstruction are shown on the left. In the last row ISAR image with full dataset is presented.

cases when no ideal sparsity is assumed and ISAR images are blurred due to uncompensated motion or nonuniform target motion during the considered CIT.

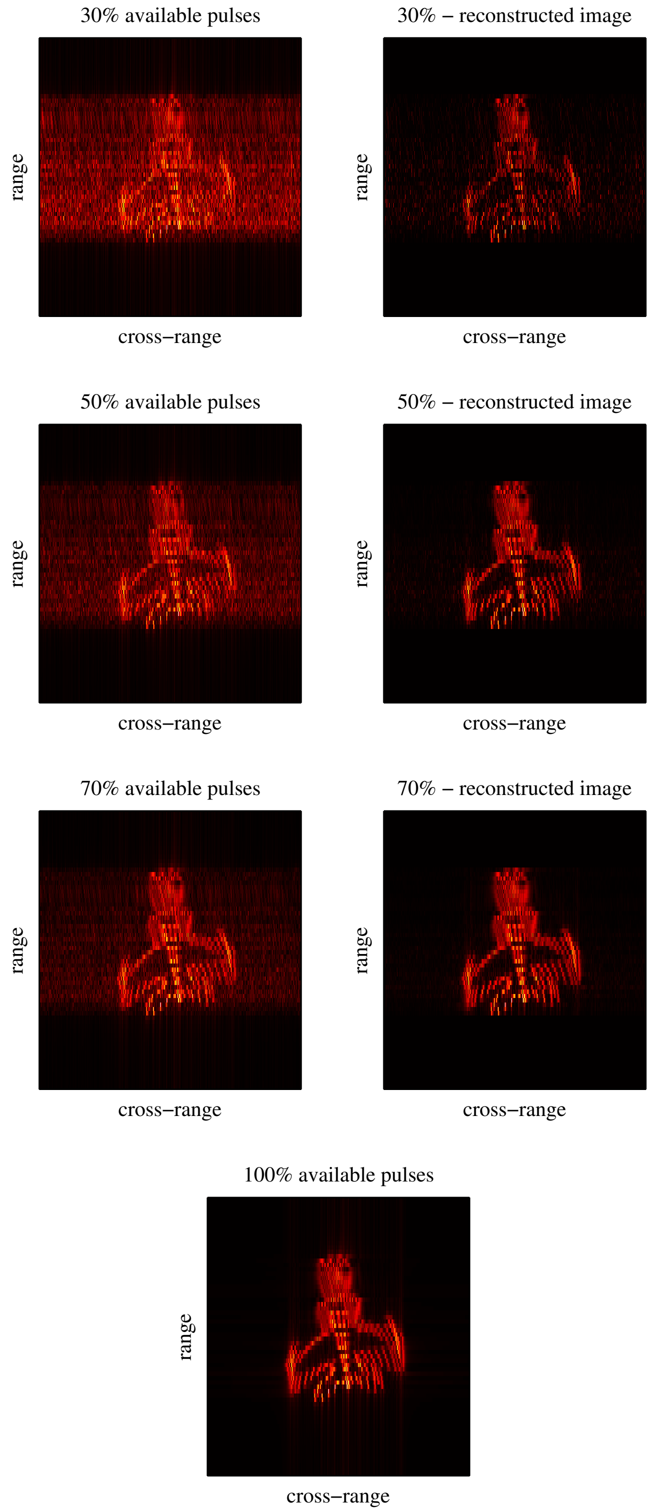


Fig. 3. Mig 25 example. This signal is only approximately sparse. The reconstruction is performed based on 30%, 50%, and 70% of the available pulses and reconstructed images are shown on the right side. Images without missing pulses reconstruction are shown on the left. In the last row ISAR image with full dataset is presented.

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