

Averaged Multiple L-Spectrogram for Analysis of Noisy Nonstationary Signals

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Abstract — A time-frequency approach for improved instantaneous frequency estimation of noisy signals has been proposed. This approach is based on the averaged multiple time-frequency distribution obtained by using L-spectrograms with different values of the scaling parameter L . The L-spectrogram with low value of L is characterized by large bias and low variance, while higher L decreases the bias and increases the variance of the instantaneous frequency estimator. The averaged L-spectrogram is considered to provide a good bias-variance compromise, leading to low MSE of the instantaneous frequency estimation in the presence of noise. The proposed approach has been tested in the experiments with different values of signal to noise ratio, showing its superior performance when compared to individual L-spectrograms.

Keywords — time-frequency signal analysis, L-spectrogram, averaged multiple distribution

1. INTRODUCTION

Time-frequency analysis has been used in various applications in which different types of non-stationary signals are encountered. For instance, time-frequency tools has been widely applied in radar and sonar systems, communications, multimedia systems, biomedical signal analysis [1]-[6], etc. Having in mind that different applications bring different requirements, various time-frequency distributions have been proposed [1], [2]. They are usually classified into linear [1], quadratic [3]-[5] and higher order distributions [6], [7]. In order to provide an accurate instantaneous frequency (IF) estimation, time-frequency distributions should produce good concentration in the time-frequency plane. Namely, the time-frequency distributions should reduce the spread factor caused by higher phase derivatives especially in the case of fast varying IF. Consequently, commonly used quadratic distributions, such as the spectrogram and the Wigner distribution, are not suitable for analysis of signals with fast varying IF. In such a

case, higher order and complex-time distributions [6], [7] have been used instead. However, by increasing the distribution order, the complexity of its realization increases. Additionally,

the signals are usually corrupted by noise, which increases the variance of the time-frequency estimator and may significantly affect the precision of IF estimation. This is in particular evident in the case of higher order distributions, which are more sensitive to noise than the quadratic distributions.

Finally, we may also observe that an efficient time-frequency distribution should provide a good trade-off between bias and variance, while keeping the realization as simple as possible [8]-[10]. Therefore, in this paper we propose the averaged multiple L-spectrogram, for the analysis of signals corrupted by Gaussian noise. The L-forms of time-frequency distributions have been proposed in order to improve the concentration of signal components [11]. By increasing the value of parameter L , the bias of the L-spectrogram decreases and the accuracy of IF estimation is improved. On the other hand, larger values of L significantly increase the variance, hence producing a large mean square error (MSE) of IF estimation in the presence of noise. Thus, to reduce the variance, lower values of L should be used. Consequently, better bias-variance trade-off can be achieved by averaging time-varying spectra obtained by using the L-spectrograms with different values of the parameter L .

The proposed approach is, in general, based on similar concepts as the multiple windows spectrograms [12]-[14], which are usually realized by using different functions such as Slepian functions, Hermite functions, etc. The Slepian functions (Thomson method) have been used for stationary spectrum estimation, while for non-smooth spectra the performance is degraded due to cross-correlation between spectra [12]. Thus, in the case of time-varying signals, the Hermite functions are used to improve the results. The number of functions and the optimal weights are usually determined by using optimization methods such as the iterative quasi-Newton algorithm [12],[13]. Furthermore, it is often necessary to calculate the weights for each windowed part of the signal [14]. Therefore, in order to simplify the calculations and to avoid the optimization algorithms, we consider the averaged multiple L-spectrogram, since

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the bias-variance trade-off can be achieved by simple averaging of the L-spectrograms, without using special window functions and weighting coefficients.

The paper is organized as follows. The time-frequency analysis based on the L-spectrogram has been reviewed in Section II. The averaged multiple L-spectrogram is proposed in Section III. The experimental results are presented in Section IV, while the concluding remarks are given in Section V.

2. TIME-FREQUENCY ANALYSIS BASED ON THE L-SPECTROGRAM

Time-frequency representations have been used to analyze time-varying spectral properties of non-stationary signals. The commonly used approaches are obtained by introducing the time dependence into the Fourier analysis using the time windowing technique. So, the short-time Fourier transform (STFT) is defined as:

$$STFT(t, \omega) = \int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j\omega\tau} d\tau, \quad (1)$$

where $x(t)$ is a signal, and $w(t)$ is a window function. The spectrogram is the energetic version of STFT and it is defined as: $SPEC(t, \omega) = |STFT(t, \omega)|^2$. The main drawback of the spectrogram is low time-frequency resolution for signals with non-constant IF.

The L-forms of time-frequency distributions have been introduced as a solution for improved concentration in the time-frequency domain. They are based on the concept of frequency linearization around a considered time instant t , provided that the value of IF at the same t remains constant. The L-spectrogram has been defined as a squared module of the L-STFT [11]:

$$SpecL(t, \omega) = \left| \int_{-\infty}^{\infty} x^L\left(t + \frac{\tau}{L}\right)w(\tau)e^{-j\omega\tau} d\tau \right|^2. \quad (2)$$

Note that, comparing to the standard spectrogram, the lag-coordinate is scaled by L , while the signal is raised to the L -th power in order to provide the appropriate IF representation. Thus, for $L=1$ the standard spectrogram is obtained, while for $L>1$, (2) represents a higher order distribution. Some properties of the L-spectrogram are analyzed in the sequel.

Resolution of the L-spectrogram: Let us assume that the signal $x(t)$ is of short duration and concentrated at $t=0$ into an interval $\Delta t \rightarrow 0$. If the window $w(t)$ is time limited to $|t| < T/2$, then $STFT_L$ is time limited to

$|t| < T/2L$, hence its time duration is $d=T/L$. Furthermore, if: $x(t)=e^{-j\omega_x t}$, then:

$$STFT_L(t, \omega) = W(\omega - \omega_x) e^{jL\omega_x t}$$

holds, where W is the Fourier transform of the window function. The width of $W(\omega)$ in the case of rectangular window is $D=4\pi/T$, and consequently, $dD=4\pi/L$. Thus, by increasing the value of L in the L-STFT, the product of durations d and D can become arbitrary small, i.e., the resolution can be arbitrary high in both directions, simultaneously.

Spreading factor of the L-spectrogram: For a signal $x(t)=Ae^{j\phi(t)}$, the time-frequency representation providing the energy distribution along the IF can be generally written as follows:

$$TFR(t, \omega) = 2\pi A^2 \delta(\omega - \phi'(t)) *_{\omega} FT\{e^{jQ(t, \tau)}\} *_{\omega} W(\omega), \quad (3)$$

where the Fourier transform is denoted by FT, while $W(\omega)$ is the Fourier transform of the window $w(t)$. The function $Q(t, \tau)$ is called a spread factor defining the distribution spread around the IF. It contains different higher order derivatives of the phase function $\phi(t)$ and it depends on the time-frequency distribution. The optimal distribution for a certain signal should be concentrated along its IF with the smallest possible spread factor. The spread factor for the L-spectrogram is given by:

$$Q(t, \tau) = \sum_{s=2}^{\infty} \frac{\phi^{(s)}(t)\tau^s}{s!L^{s-1}}. \quad (4)$$

We might observe that for larger values of L , the influence of higher phase derivatives significantly decreases, hence reducing the bias. However, at the same time, the variance of IF estimation increases, since the signal and noise are raised to power of L .

Signal to noise ratio: In the presence of noise, the signal on the L -th power can be approximated as:

$$x^L(t) = (s(t) + \varepsilon(t))^L \approx s^L(t) + Ls^{L-1}(t)\varepsilon(t) = s^L(t) + \varepsilon_L(t). \quad (5)$$

The above approximation holds under the assumption that the noise is small with respect to the signal, i.e., σ_{ε}/A is less than 1, where A is the amplitude of signal $s(t)$, while σ_{ε}^2 is the variance of the noise $\varepsilon(t)$. The noise to signal ratio in this case is L^2 times higher than the original noise to signal ratio:

$$NSR = \frac{L^2 \sigma_{\varepsilon}^2}{A^2}. \quad (6)$$

3. AVERAGED MULTIPLE L-SPECTROGRAM

In order to provide good compromise between low bias and low variance requirements, when the signal is corrupted by additive noise, the averaged multiple L-spectrogram can be defined as follows:

$$MSpecL(t, \omega) = \frac{1}{K} \sum_{L=1}^K SpecL(t, \omega), \quad (7)$$

where K is the number of employed L-spectrograms. It has been shown experimentally that $K=6$ provides satisfactory IF estimation (in terms of the low estimation error) for a fast-varying phase function. Note that by using higher K fast IF variations could be followed more precisely. However, under the noise influence, the $SpecL$ for $L>6$ might introduce spurious noisy peaks, which would significantly degrade the IF estimation precision. Hence, $K=6$ has been chosen as an optimal case for all tested signal. The IF is estimated as:

$$\hat{\omega}(t) = \arg \left[\max_{\omega \in Q} (MSpecL(t, \omega)) \right], \quad (8)$$

where $Q = \{\omega: 0 \leq |\omega| < \pi/(2T)\}$, represents the basic interval along the frequency axis.

The L-spectrograms in the cases $L=1$ and $L=2$ are characterized by large bias, while $L=4$ and $L=5$, have low bias, but quite high variance. The variance of $MspecL$ (for $K=6$) has been measured experimentally through simulations, as it has been also analyzed in [8], and the results are provided in Section 4. It has been shown that the variance of $MspecL$ tends toward lower variances (i.e. toward $\text{var}\{SpecL\}$ for $L<3$), while the bias of $MspecL$ tends toward lower biases (i.e., toward $\text{bias}\{SpecL\}$, $L=4$). The expressions for bias and variance of the $MspecL$ can be approximated as:

$$\begin{aligned} bias &= E\{MSpecL\} = MSpecL^s + N\sigma_v^2 \frac{1}{K} \sum_{i=1}^K L_i^2, \\ var &= \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K cov\{SpecL_i, SpecL_k\} < \\ &< \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K 2|STFT_{L_i}^s|^2 |STFT_{L_k}^s|^2 + \\ &\frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K 2|STFT_{L_i}^s STFT_{L_k}^s| L_i L_k N\sigma_v^2 + L_i^2 L_k^2 N^2 \sigma_v^4, \end{aligned} \quad (9)$$

where s in the superscript of $MSpecL^s$ denotes non-noisy signal. This approximate variance form is derived in the Appendix. When analyzing the terms that reflect the noise influence in (10), we have found that the second term is approximately less than $2N\sigma_v^2 |STFT_L^s|^2 L^2$ for

$L=3.5$ and $K=6$, while the third term is approximately less than $N^2\sigma_v^4 L^4$ for $L=3.8$. Since L should be an integer, the above observations are said to be true for $L=3$. We will emphasize again that the worst case (rough) approximations have been used, and thus, we may say that the analysis corresponds to the experimental results.

3.1. Realization for multicomponent signals

The cross-terms free version of L-spectrograms (for $L>1$) is calculated by firstly separating signal components, as follows:

- For each time instant we find the position of the STFT maximum:

$$\omega_1(t) = \arg \left\{ \max_{\omega} STFT(t, \omega, \frac{\tau}{L}) \right\}, \text{ where}$$

$$STFT(t, \omega, \frac{\tau}{L}) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{L}) w_L(\tau) e^{-j\omega\tau} d\tau.$$

The corresponding signal component is obtained as:

$$x_1(t) = \int_{\omega_1 - \Delta\omega}^{\omega_1 + \Delta\omega} STFT(t, \omega, \frac{\tau}{L}) e^{j\omega\tau} d\omega.$$

- Set $STFT(t, \omega, \frac{\tau}{L}) = 0$, for $[\omega_1 - \Delta\omega, \omega_1 + \Delta\omega]$, and repeat the procedure P times, where P is the expected number of components.

Hence, at each time point, $x_i(t)$ will contain only one of the signal components, and thus the L-spectrogram can be calculated for $x_i(t)$ without producing the cross-terms. It is important to observe that the cross-terms will be completely avoided if the distance between components is at least $2\Delta\omega$. Note that the width $2\Delta\omega$ can be adjusted to each signal component according to the realization given in [15]. In the case when the number of components is not known a priori, after removing the region around the STFT maximum, we should determine next $\max\{STFT(t, \omega, \tau/L)\}$ for remaining part of frequency points ω . If it is higher than an assumed level for a signal component, we continue the procedure described above; otherwise the procedure is finished.

4. EXPERIMENTAL RESULTS

4.1. Example 1 (10)

Consider the signal in the form: $x_1(t) = Ae^{j(3/4\cos(4\pi t) + 2/3\cos(\pi t))} + \varepsilon(t)$. The averaged multiple L-spectrogram is calculated for $K=6$ ($L=1, \dots, 6$). The individual L-spectrograms are shown in Fig. 1.a-f, respectively, while the averaged multiple L-spectrogram is shown in Fig. 1.g.

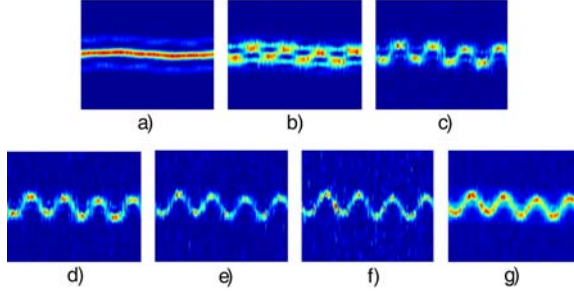


Fig. 1. a) *SpecL*, $L=1$, b) *SpecL*, $L=2$, c) *SpecL*, $L=3$, d) *SpecL*, $L=4$, e) *SpecL*, $L=5$, f) *SpecL*, $L=6$, g) *MSpecL* (SNR=15dB)

The variances for *MSpecL* and *SpecL* (for $L=1,2$ and 3) are shown in Fig. 2, while the variances of *MSpecL* and *SpecL* ($L=4,5$ and 6) are compared in Fig. 3. The variance of *MSpecL* is lower than the variance of *SpecL* for $L=3$, and thus we may say that it tends toward lower variances. On the other hand, the bias of *MSpecL* corresponds to the bias of *SpecL* for $L=4$ (it tends toward lower biases). The MSEs of IF estimation for individual L -spectrograms and the *MSpecL* are given in Table 1. Note that the *MSpecL* provides significantly lower MSE than any of the individual L -spectrograms.

TABLE 1: MSE OF IF ESTIMATION FOR L -SPECTROGRAMS FOR DIFFERENT VALUES OF L AND FOR THE PROPOSED *MSpecL*

	<i>SpecL</i> , $L=1$	<i>SpecL</i> , $L=2$	<i>SpecL</i> , $L=3$	<i>SpecL</i> , $L=4$	<i>SpecL</i> , $L=5$	<i>SpecL</i> , $L=6$	<i>MSpecL</i>
MSE	47	26	8.9	6.1	3.8	4.4	0.83

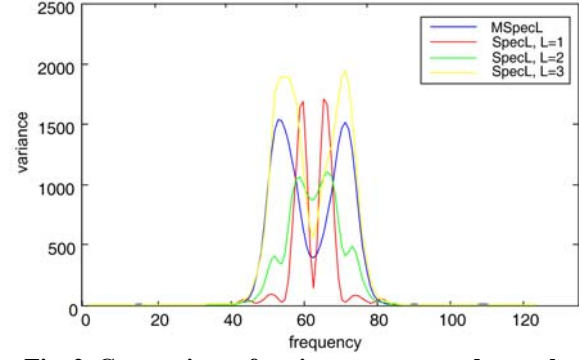


Fig. 2. Comparison of variances, measured at each frequency point ($L=1, L=2$ and $L=3$)

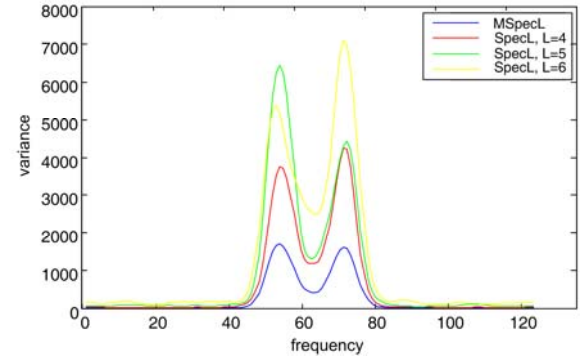


Fig. 3. Comparison of variances, measured at each frequency point ($L=4, L=5$ and $L=6$)

Furthermore, the efficiency of the proposed averaged multiple L -spectrogram has been tested for different values of SNR. From Table 2, it can be observed that the *MSpecL* provides low MSE, which is again quite lower than the MSE achieved by the *SpecL* for different L .

TABLE 2: MSE OF IF ESTIMATION FOR L -SPECTROGRAMS ($L=1,2,3,4,5$, AND 6) AND FOR THE PROPOSED *MSpecL* (FOR THE SIGNAL $x_1(t)$)

SNR	20dB	16dB	13dB	10dB
<i>SpecL</i> , $L=1$	47.5	48	48	47
<i>SpecL</i> , $L=2$	24.4	30.5	30	38
<i>SpecL</i> , $L=3$	9	11.5	20.6	22
<i>SpecL</i> , $L=4$	3	6.1	44	69
<i>SpecL</i> , $L=5$	2.6	5.3	57	276
<i>SpecL</i> , $L=6$	2.5	3	150	538
<i>MSpecL</i>	0.6	0.7	1.6	9

4.2. Example 2

In order to demonstrate that the proposed method can be useful even for signals with fast IF variations within a few signal samples, we consider a signal:

$$x_2(t) = Ae^{j(\cos(1.5\pi t) + 1/2 \cos(3\pi t) + 1/2.5 \cos(5\pi t))} + \varepsilon(t)$$

From Figs. 4 and 5, it can be seen that the multiple L -spectrogram results in poorer resolution (higher bias) when compared to the L -spectrograms for $L=4, L=5$ and $L=6$. However, the variance is lower than the variance for $L=4,5,6$, this being due to the benefit gained from averaging (Fig. 5). The resulting representation *MSpecL*

is smoother (lower variance). The MSEs of IF estimation are summarized in Table 3.

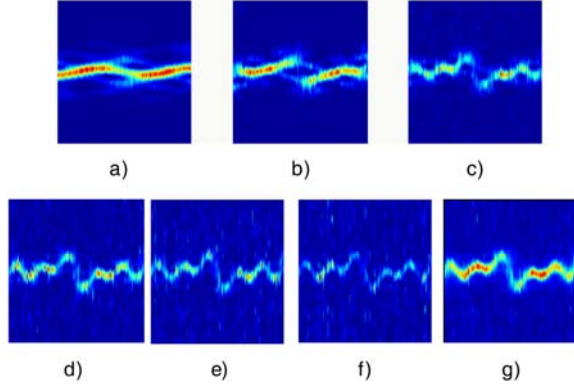


Fig. 4. a) *SpecL*, $L=1$, b) *SpecL*, $L=2$, c) *SpecL*, $L=3$, d) *SpecL*, $L=4$, e) *SpecL*, $L=5$, f) *SpecL*, $L=6$, g) *MSpecL* (SNR=15dB), for the signal $x_2(t)$

TABLE 3: MSE OF IF ESTIMATION FOR THE SIGNAL $x_2(t)$

	<i>SpecL</i> , $L=1$	<i>SpecL</i> , $L=2$	<i>SpecL</i> , $L=3$	<i>SpecL</i> , $L=4$	<i>SpecL</i> , $L=5$	<i>SpecL</i> , $L=6$	<i>MSpecL</i>
MSE	28.2	17.4	7.9	5.6	12	9.8	2.5

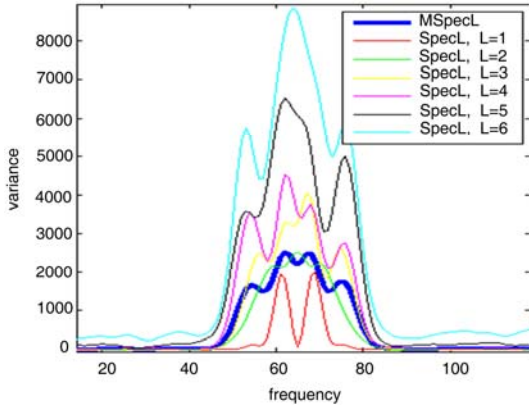


Fig. 5. The estimated variances for *MSpecL* and *SpecL* for $L=1,2,3,4,5,6$

Similar results are obtained for the multicomponent signals whose components do not intersect. The components are firstly separated to avoid the presence of cross-terms, as described in Section 3.1. Each component is further processed separately, in the same way as in the previous examples. The results are shown in Fig. 6.

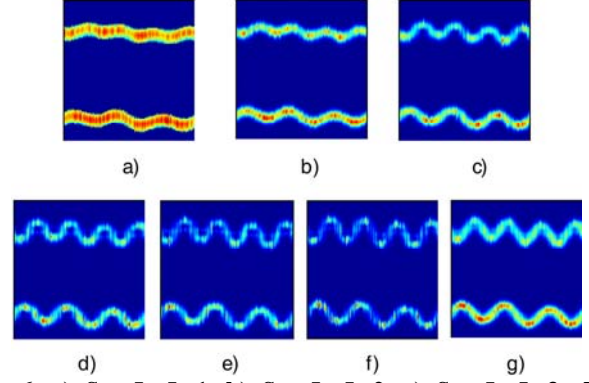


Fig. 6. a) *SpecL*, $L=1$, b) *SpecL*, $L=2$, c) *SpecL*, $L=3$, d) *SpecL*, $L=4$, e) *SpecL*, $L=5$, f) *SpecL*, $L=6$, g) *MSpecL* (SNR=15dB), for the signal $x_2(t)$

4.3 Example 3

Additionally, the performance of *MSpecL* is tested for radar signal, describing the Micro-Doppler signature of human leg movements during walking. For the realization of L-spectrograms the signal is oversampled, i.e., interpolated by 60 samples (the least common multiple of $L=1, \dots, 6$). Different L-spectrograms and the proposed *MSpecL* are shown in Fig. 7. The signal is corrupted by the Gaussian noise with SNR=10dB. We may observe that by increasing L , the concentration increases, but at the cost of higher noise level. The *MSpecL* produces concentrated representation with significantly lower noise influence due to averaging.

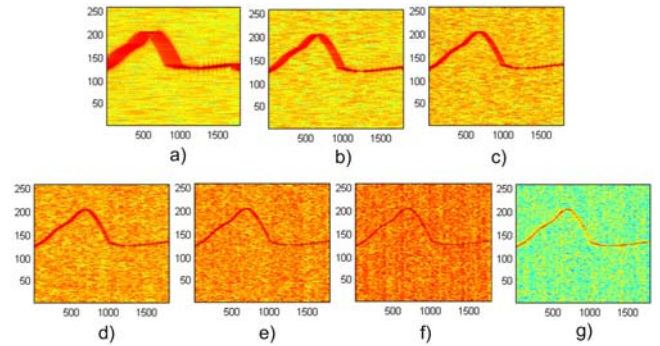


Fig. 7. a) *SpecL*, $L=1$, b) *SpecL*, $L=2$, c) *SpecL*, $L=3$, d) *SpecL*, $L=4$, e) *SpecL*, $L=5$, f) *SpecL*, $L=6$, g) *MSpecL*

5. CONCLUSION

An efficient time-frequency distribution based on the averaged L-spectrogram is proposed to improve the accuracy of fast varying IF estimation of noisy signals. The presented approach yields low bias and low variance of the estimator in the presence of additive Gaussian noise. Thus, it results into more accurate IF estimates

than each of the individual L-spectrograms. The low MSEs of IF estimation are achieved even for noisy signals with fast phase variations.

APPENDIX

The variance of the $MspecL$ can be calculated as:

$$\begin{aligned} var &= \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K cov\{Spec_{L_i}, Spec_{L_k}\} = \\ &= \sum_{L_i=1}^K \sum_{L_k=1}^K \left| E\{STFT_{L_i}^* STFT_{L_k}\} \right|^2 + \left| E\{STFT_{L_i} STFT_{L_k}\} \right|^2 \end{aligned} \quad (11)$$

After some mathematical manipulations and having in mind that for complex white Gaussian noise we have: $E\{v(i_1)v(i_2)\}=0$, $E\{v^*(i_1)v^*(i_2)\}=0$, the above expression becomes:

$$\begin{aligned} var &= \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K \left| STFT_{L_i}^s STFT_{L_k}^{*s} + L_i L_k \sigma_v^2 \Xi \right|^2 \\ &+ \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K \left| STFT_{L_i}^s STFT_{L_k}^s \right|^2 \end{aligned} \quad (12)$$

where:

$$\Xi = \sum_{m_1=1}^{N-1} \sum_{m_2=1}^{N-1} w(m_1, m_2) s^{L_i-1} \left(n + \frac{m_1}{L_i}\right) s^{*(L_k-1)} \left(n + \frac{m_2}{L_k}\right) e^{-j\frac{2\pi}{N}k(m_1-m_2)}$$

Here it is assumed that $x^L(t) \approx s^L(t) + Ls^{L-1}(t)v(t)$ and $w(m_1, m_2) = w(m_1)w(m_2)$.

Furthermore, by considering the rectangular window w and the signal with unit amplitude, a rough approximation of the variance is obtained as:

$$\begin{aligned} var &< \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K 2 \left| STFT_{L_i}^s \right|^2 \left| STFT_{L_k}^s \right|^2 \\ &+ \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K 2 \left| STFT_{L_i}^s STFT_{L_k}^s \right| L_i L_k N \sigma_v^2 \\ &+ \frac{1}{K^2} \sum_{L_i=1}^K \sum_{L_k=1}^K L_i^2 L_k^2 N^2 \sigma_v^4. \end{aligned} \quad (13)$$

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