An Automated Signal Reconstruction Method based on Analysis of Compressive Sensed Signals in Noisy Environment

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[1]***Abstract- An analysis of signal reconstruction possibility using a small set of samples corrupted by noise is considered. False detection and/or misdetection of sparse signal components may occur as a twofold influence of noise: one is a consequence of missing samples, while the other appears from an external source. This analysis allows us to determine a minimal number of available samples required for a non-iterative reconstruction. Namely, using a predefined probability of error, it is possible to define a general threshold that separates signal components from spectral noise. In the cases when some components are masked by noise, this threshold can be iteratively updated. It will render that all components are detected, providing an iterative version of blind and simple compressive sensing reconstruction algorithm.***

Index Terms – compressive sensing, missing samples, noise, signal reconstruction algorithms

1. Introduction

In the last few years, compressive sensing (CS) has attracted a significant attention of researchers, since it provides successful signal reconstruction using incomplete set of randomly selected measurements [1]-[4]. The CS deals with signals sampled at rates lower than those required by the Shannon-Nyquist theorem. Beside the missing samples appearing due to the lower (and random) sampling rate, we might also deal with missing samples due to the removal of unwanted signal parts as it is done in robust statistics. In order to successfully apply the CS algorithms, the signal has to be sparse in a certain transform domain [5]-[6]. Sparsity is very important property that has been used in many applications, such as the reconstruction of biomedical images [7], radars [8], multimedia [14], etc. Also, the sparsity strategy is employed in optical systems for information encoding, where sparse data are used in the phase-modulated cryptosystems in order to achieve higher security of the optical authentication method [9]. There are many applications dealing with signals that are sparse in time-frequency domain: communications signals (e.g., frequency hopping modulation scheme), acoustic signals, slowly varying chirps which are used in many radar applications and geophysics, etc.

The signal reconstruction is usually done by using the basis pursuit (BP) method or greedy algorithms as more efficient alternative, e.g. thresholding and orthogonal matching pursuit (OMP) [10]-[17]. For instance, the iterative hard thresholding algorithm (IHT) is used to solve a K sparse problem as follows: $x^{[n+1]}=H_K(x^{[n]}+\Phi^T(y-\Phi x^{[n]}))$, where y represents a set of measurements, Φ is a measurement matrix, $x^{[0]}=0$, while $x^{[n+1]}$ is reconstructed signal after the n-th iteration. Also, H_K (***a***) is the non-linear operator that sets all but the largest (in magnitude) K elements of ***a*** to

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zero. The convergence of this algorithm is guaranteed under the condition $\|\Phi\|_2 < 1$. In practice, in many cases for compressible signals, it is not uncommon to lose the convergence and be unstable. This is often due to the selection of wrong coefficients in the hard-thresholding step [18]. The OMP selects, in each iteration, one column of measurement matrix that has the highest correlation with the residual part of measurement vector. Then, we subtract its contribution from the measurements and iterate on the residual. In some cases, these methods outperform BP [17], and demands lower computational complexity than BP. Generally, it is difficult to accurately determine the minimal number of samples needed for successful signal reconstruction. The situation is even more complicated when the available samples are affected by noise [19]-[22]. In this case, the signal reconstruction from noisy measurements depends also on the support recovery, which deals with the problem of estimation of signal components positions. An interesting approach which assumes a priori knowledge of the partial signal support has been analyzed in [23].

In this paper we propose an approach to determine the exact number of samples needed for an accurate signal reconstruction in the presence of noise. Conceptually, the goal is to model the noise appearing as a consequence of missing samples, such that we can control the noise level in terms of missing samples number. In order to produce a practical reconstruction method starting from the conceptual one, we will focus on the frequency domain (e.g., DFT) as a domain of sparsity. However, the same concept could be applied in other domains as well. Furthermore, the problem of an external additive Gaussian noise influence is considered. In this case, for a given signal to noise ratio (SNR), an expression that provides the number of samples required for CS reconstruction is derived. It ensures, with an arbitrary predefined probability, that the signal components will be detected within the noise. In that sense, based on the number of available samples, it is possible to set a threshold which will distinguish between signal components and noise. Consequently, this analysis allows us to reconstruct all components within a single iteration of the reconstruction algorithm. Furthermore, if we deal with critically low number of samples, some of the components will be masked by noise. In order to avoid components misdetection, it is necessary to remove the contribution of previously detected components and to repeat procedure using the updated threshold. The state of art thresholding method [14], usually assumes that the number of required non-zeros is known, or the user needs to specify a priori the error floor value and to select components as long as the error is below the floor. In the proposed case the threshold is used for blind components detection (no a priori knowledge is assumed). It can be set adaptively depending on the number of available samples and external noise influence. In this way, we provide a blind reconstruction procedure for an unknown number of components, although some of them are masked by noise. The theoretical considerations are verified by numerical examples.

The paper is structured as follows. After the Introduction, the theoretical analysis of missing samples effects in the presence of external Gaussian noise is considered in Section II. The blind CS reconstruction algorithms (single- and multi- iterations based) is proposed in Section III. In Section IV, the theory is proved on the numerical examples. The concluding remarks are given in the last section.

2. Theoretical Background

Observe a signal that consists of K sinusoidal components in the form:

$$s(n)=\sum_{i=1}^{K}A_i e^{j2\pi k_i n/N} \tag{1}$$

where A_i and k_i denote amplitude and frequency of the i-th signal components, respectively. The discrete Fourier transform (DFT) of such a signal can be written as:

$$S(k)=N\cdot\sum_{n=1}^{N}\sum_{i=1}^{K}A_i e^{-j2\pi(k-k_i)n/N} \tag{2}$$

Consider now the set of samples:

$$\Omega=\{x(1),x(2),...,x(N)\}=\{\sum_{i=1}^{K}A_i\exp(-j2\pi(k-k_i)n/N),\ n=0,1,...,N-1\}, \tag{3}$$

where the number of samples N is determined by the sampling theorem. A subset $\Theta\subset\Omega$ is obtained by randomly selecting M out of N measurements: $\Theta=\{y(1),y(2),...,y(M)\}$. As a consequence of missing samples, a noise appears in spectral domain ruining the signal sparsity [22], [24]. For instance, let us observe a simple signal case with one component A_1=1, k=k_1: $x(n)=e^{-j2\pi(k-k_1)n/N}$. The DFT over available set of samples can be written as follows:

$$F(k)=\sum_{n=1}^{M}y(n)=\sum_{n=1}^{N}\{x(n)-\varepsilon(n)\}, \tag{4}$$

where at the positions of missing samples, the noise can be modeled as:

$$\varepsilon(n)=x(n)=e^{-j2\pi(k-k_1)n/N}, \tag{5}$$

while $\varepsilon(n)$=0 otherwise. Thus, $\varepsilon(n)$ contains a set of (N-M) missing values. Obviously, $E\{F_{k=k_1}\}=M$, $E\{F_{k\neq k_1}\}=0$. Now, the variance of noise in F can be calculated as follows:

$$\begin{aligned}\mathrm{var}(F_{k\neq k_1})&=E\{[y(1)+...+y(M)]\cdot[y(1)+...+y(M)]^*\}=\\&=M\cdot E\{x(n)x^*(n)\}+M(M-1)\cdot E\{x(n)x^*(m)\}=\\&=M\cdot 1+M(M-1)\frac{-1}{N-1}=M\frac{N-M}{N-1}\end{aligned}$$

where $(\cdot)^*$ denotes complex conjugate. Here, we have applied the following equalities:

$$\sum_{n=1}^{N}E\{x(n)x^*(m)\}=0, \tag{6}$$

$$E\{x(n)x^*(n)\}=1,\quad E\{x(n)x^*(m)\}_{n\neq m}=\frac{-1}{N-1}, \tag{7}$$

Furthermore, let assume that the signal is sparse in the DFT domain and consists of K components [22]. The variance of spectral noise caused by missing samples is:

$$\sigma^2{}_{MS} = \text{var}\{F_{k\neq k_i}\} = M\,\frac{N-M}{N-1}\sum_{i=1}^{K} A_i{}^2 \,. \tag{8}$$

It is important to emphasize that noise produced by the missing samples, in certain cases, may exceed the value of signal component. The error may appear if any of the DFT values at noise-alone positions is higher than the DFT value of signal component, since it will lead to false component detection. In the sequel, we derive the expression for the probability of error on the basis of the spectral noise variance. According to the central limit theorem, the real and imaginary parts of noise-alone DFT values can be described by the Gaussian distribution. Then the probability density function for the absolute DFT values at noise-alone positions is Rayleigh-distributed. The pdf for the absolute DFT values at noise-alone positions are Rayleigh-distributed [22]. Hence, the probability of false detection of the *i*-th component (probability that all *N-K* noise-alone DFT values are higher than the *i*-th component amplitude MA_i) is given by:

$$P_{err}^{i} \cong 1-\left[1-\exp\left(-\frac{M^2 A_i{}^2}{\sigma_{MS}{}^2}\right)\right]^{N-K}, \tag{9}$$

where i=1,…,K. Hence, for given probability of error, it is easy to determine the exact number of samples M, which will ensure no false component detection at noise-alone positions. Note that the previous equations assume measurements that are not affected by external noise. Since the noisy environment is more interesting for practical applications, we further assume that the signal is corrupted by Gaussian noise with variance σ_N^2. In that case, we need to consider the total variance of disturbance caused by both the external noise and noise due to the missing samples (involved random variables are uncorrelated):

$$\sigma^2 = \sigma_{MS}^2 + M\sigma_N^2 = M\,\frac{N-M}{N-1}(A_1^2+...+A_K^2) + M\sigma_N^2 . \tag{10}$$

Obviously, if we want to keep the same probability of error as in the noiseless case, we need to increase the number of measurements, i.e., we need to satisfy the following condition:

$$\frac{\sigma_{MS}^2}{\sigma^2} = \frac{M\,\dfrac{N-M}{N-1}(A_1^2+A_2^2+...+A_K^2)}{M_N\,\dfrac{N-M_N}{N-1}(A_1^2+A_2^2+...+A_K^2)+M_N\sigma_N^2} = 1 . \tag{11}$$

Here, M_N denotes the increased number of measurements required in the presence of external noise. Having in mind that SNR is given by:

$$SNR = \sum\nolimits_{i=1}^{K} A_i^2 / \sigma_N^2 \,, \tag{12}$$

we have:

$$\frac{M(N-M)}{M_N}\,\frac{SNR}{SNR(N-M_N)+(N-1)} = 1 . \tag{13}$$

Now, for given N, M and SNR, the number of available noisy samples M_N follows as a solution of the equation:

$$M_N{}^2\cdot SNR - M_N(SNR\cdot N + N - 1) + SNR\cdot(MN - M^2) = 0 . \tag{14}$$

For example, let observe the case N=256 and M=192, while SNR=10dB is assumed. According to the previous relation, the number of available samples must be increased to M_N=227.

3. ALGORITHM FOR CS RECONSTRUCTION

In order to provide an efficient signal reconstruction, it is important to separate components that originate from signal from those belonging to the noise. In that sense, we can define the probability that a noise sample in frequency domain (a DFT value at noise-alone position) is below a threshold T:

$$Q(T)=1-\int_{T}^{\infty}\frac{2\zeta}{\sigma^2}e^{-\frac{\zeta^2}{\sigma^2}}d\zeta=1-e^{-\frac{T^2}{\sigma^2}}. \tag{15}$$

The probability that (N-K) values corresponding to noise are lower than T is:

$$P(T)=\left(1-\exp(-\frac{T^2}{\sigma^2})\right)^{N-K}. \tag{16}$$

For instance, we can assume $P(T)$=0.99 and calculate the threshold as follows:

$$T=\sqrt{-\sigma^2\log(1-P(T)^{\frac{1}{N-K}})}. \tag{17}$$

Observe that the number of components K is usually $K<<N$, and without loss of generality we can neglect the influence of K in (17). In this way, we can define the blind CS reconstruction procedure which does not require the a priori knowledge of components number in the sparsity domain. Assume that $K_p \le K$ components greater than T are detected. After determining their positions, we compute the exact amplitude values of these components, by using the set of linear equations:

$$\sum_{i=1}^{K_p}X(k_i)\exp(j2\pi k_i n_a)=x(n_a), \tag{18}$$

for each available sample n_a. Therefore, in the case that all signal components are above noise level in DFT, a simple single-iteration CS reconstruction algorithm can be defined. This algorithm is based on the analysis of Constant False Alarm Rate (CFAR) used to set the threshold and to detect components embedded in noise. The threshold depends on the global signal energy (used to calculate σ_{MS}) and the estimated external noise variance σ_N. The noise variance can be estimated using median of real and imaginary parts of the signal, as it was done in [3] (pages 420,421).

Algorithm 1 –Automated Single-pass Solution

1. **Calculate** the noise threshold T for a given probability $P(T)$ and σ (the threshold does not depend on the number of components K):

   $$T=\sqrt{-\sigma^2\log(1-P(T)^{\frac{1}{N}})};$$

2. **Calculate** the initial DFT vector **X** that corresponds to the set of available measurements.
3. **Find** vector of positions of DFT components higher than T:

$$\mathbf{k} = \arg\left\{ |\mathbf{X}| > \frac{T}{N} \right\}.$$

4. The exact DFT values at positions **k** are obtained as a solution of CS problem: $\mathbf{A_{cs}X=y}$, where the CS matrix **Acs** is obtained from the DFT matrix, using columns that correspond to the frequencies **k** and rows corresponding to *M* available measurements (where length(**k**)<*M*). The system is solved in the least square sense as:

$$\mathbf{X}=(\mathbf{Acs}^*\mathbf{Acs})^{-1}\mathbf{Acs}^*\mathbf{y},$$

where Hermitian matrix is denoted by $\mathbf{Acs}^*$. The resulting vector **X** contains the exact DFT values at positions **k**, while the remaining DFT values are zero. An efficient application of the proposed simple solution assumes unknown number of signal components with amplitudes that are above the resulting noise. Otherwise, the smallest components will not be detected. This issue is directly related to the number of available samples *M*. Namely, an approximate expression for the probability of components misdetection is given by:

$$P_{err}(M) \cong 1 - \left[1 - \exp\left(-\frac{(MA_{\min} - \sigma_{rest})^2}{\sigma_{MS}{}^2 + \sigma_N{}^2} \right) \right]^N \tag{19}$$

where
$$\sigma^2_{rest}(M) = \frac{M(N-M)}{(N-1)}(A^2 - A^2_{\min}), \tag{20}$$

while A_{min} represents the expected minimal component amplitude. Then the optimal *M* can be obtained as: $M_{opt} \geq \arg\min\{P_{err}(M)\}$, for a desired P_{err}. In other words, the idea is to calculate P_{err} for different *M* and expected A_{min}, and to choose *M* that corresponds to desired P_{err} as given in Fig.1. In this illustration, a four component signal with A_1=4.5, A_2=4, A_3=2, A_4=1.75 is observed (with additional Gaussian noise with σ^2_N=4). In the considered case, A_{min}=1.75 and from Fig.1, we can observe that for $P_{err}=10^{-2}$ we need at least *M*=180 measurements.

Furthermore, in order to provide experimental evaluation, let us observe values of M_{opt} that correspond to $P_{err}=10^{-2}$. The theoretical and experimental probabilities of components misdetection are given in Table 1. The experimental statistical results are obtained for 10000 realizations with randomly positioned available samples. We might observe that there is a high matching between the theoretical and experimental results.

TABLE 1. COMPARISON: THEORETICAL AND EXPERIMENTAL RESULTS

Comp.	P_{err} Experimentally	P_{err} Theoretically
A_1	**M_{opt}=33**	**M_{opt}=33**
	$1.7\cdot10^{-2}$	10^{-2}
A_2	**M_{opt}=43**	**M_{opt}=43**
	$1.5\cdot10^{-2}$	10^{-2}
A_3	**M_{opt}=146**	**M_{opt}=146**
	$4\cdot10^{-2}$	10^{-2}
A_4	**M_{opt}=180**	**M_{opt}=180**
	$3\cdot10^{-2}$	10^{-2}

The computational complexity of the proposed algorithm is: matrix and vector multiplication O(MK+MK^2) + matrix inversion O(K^3)=O(NM+MK+MK^2+K^3), which is in total O(NM). Note that the computational complexity for the OMP algorithm is O(KMN).

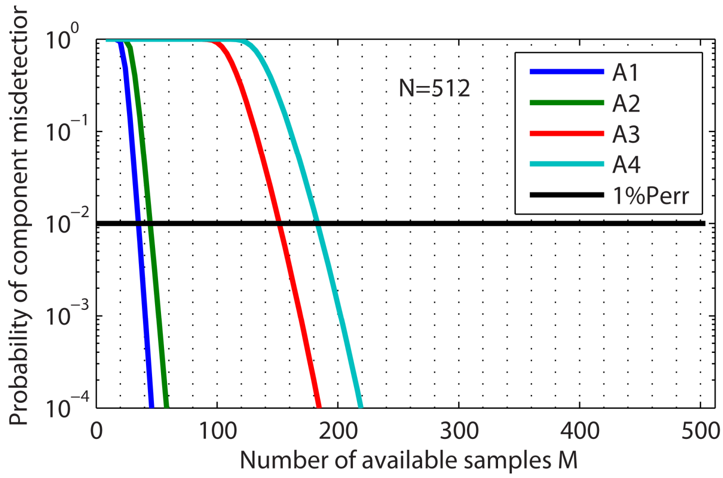

Fig. 1. Probability of components misdetection in terms of the number of available measurements

Algorithm 2 –Extended iterative solution

In the case when some components are masked by a strong noise, we need to employ a modified iterative form of the algorithm, which is summarized below. It actually needs to remove the contribution of stronger components and to reveal the smaller ones. Here, in each iteration i, a set of components on positions $\mathbf{k_i}$ will be detected and after just a couple of iterations, we are able to obtain all signal components.

Set i=1, $\boldsymbol{y}$ is a measurement vector obtained as y=x($\boldsymbol{\theta}$), where $x(n)=\sum_{l=1}^{K} A_k e^{j2\pi ln/N}$ and $\boldsymbol{\theta}=\{n_1,n_2,\ldots,n_M\}$**.** Also $\mathbf{p}=\varnothing$.

1. **Calculate** the noise threshold T for a given $P(T)$: $T=\sqrt{-\sigma^2 \log(1-P(T)^{\frac{1}{N}})}$.
2. **Calculate** the initial DFT vector $\mathbf{X}_i$ that corresponds to the available set of measurements $\boldsymbol{y}$
3. **Find** positions $\mathbf{k_i}$ in DFT vector that are greater than T:

$$\mathbf{k_i}=\arg\left\{|\mathbf{X}|>\frac{T}{N}\right\}$$

Update $\mathbf{p}=\mathbf{p}\cup\mathbf{k}_i$. The number of elements in $\mathbf{k_i}$ should be smaller than M.

4. **Solve:** $\mathbf{X}_i=\left(\mathbf{A}cs_i^*\mathbf{A}cs_i\right)^{-1}\mathbf{A}cs_i^*\mathbf{y}$, where CS matrix $\mathbf{A}cs_i^*$ contains rows defined by $\mathbf{k_i}$ set and M columns of the DFT matrix.
5. **Calculate** modified measurement vector $\boldsymbol{y}$ by subtracting the contribution of $\mathbf{X}(\mathbf{p})$:

$$\textbf{for } \forall p\in \mathbf{p}:\quad \boldsymbol{y}=\boldsymbol{x}(\boldsymbol{\theta})-X(p)\exp(j2\pi p\boldsymbol{\theta}/N);$$

Calculate new DFT vector $\mathbf{X}$ for the modified measurement vector $\boldsymbol{y}$.

6. **Update** $A^2 = \sum |\mathbf{y}|^2 / M$ **and** $\sigma_{MS}^2 = \dfrac{MA^2(N-M)}{N-1}$. **If** $A^2 < \sigma_N$ **break.**

7. **Set** i=i+1, go to 1.

The flowcharts of Algorithm 1 and Algorithm 2 are given in Fig. 2a and 2b, respectively.

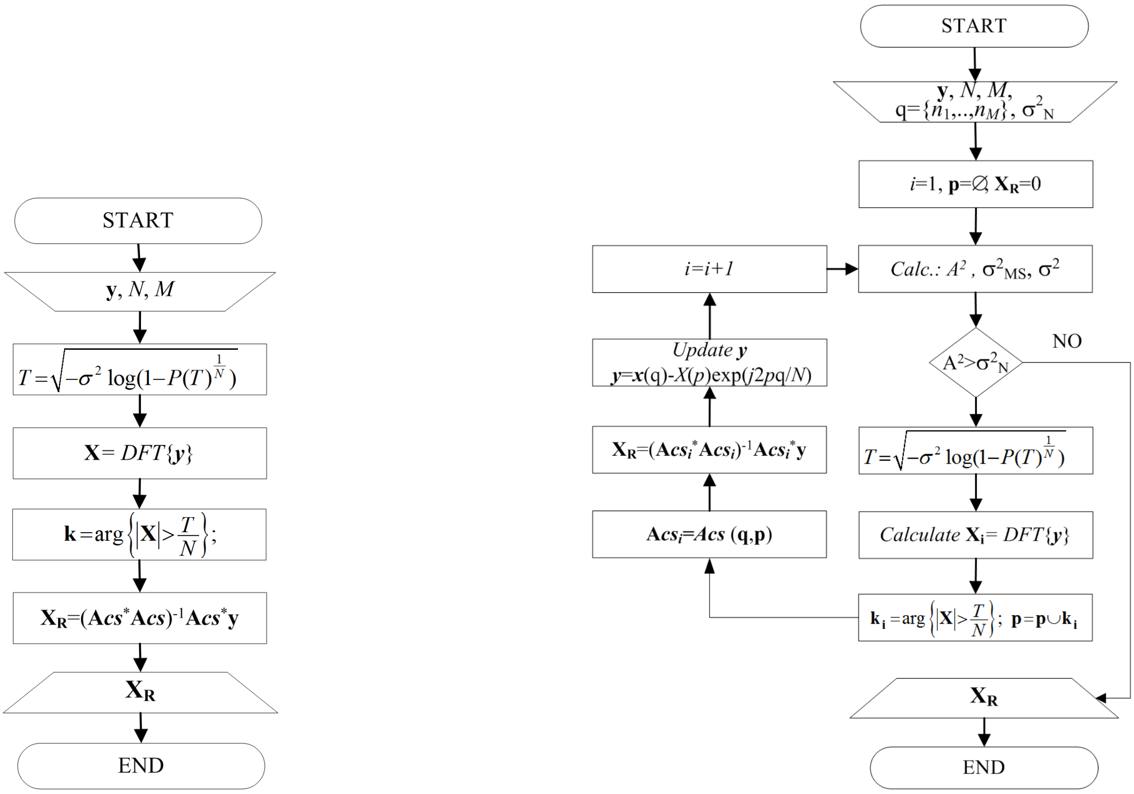

Fig. 2. Flowcharts: a) Algorithm 1, b) Algorithm 2

Accuracy: The starting assumpiton in the proposed approach is the presence of external noise in the original full data set. As such, the noise is observed in the sense that it can cause false alarms and misdetection during the components detection process. Consequently, the proposed method sets the condition based on the false alarm rate that allows us to avoid the influence of noise on signal components detection. Regarding the accuracy of the final result, in the sequel we show that the proposed method significantly reduces the initial SNR providing the denoised version of the original full signal. For the number of components, number of measurements and size of the full data *K*, *M* and *N* respectively, the input SNR of the full data set would be $SNR_i = 10\log(A_1^2 + ... + A_K^2)/\sigma_v^2$ where the energy of signal is $E\{|x(n)|^2\} = A_1^2 + ... + A_K^2$, while the noise energy corresponds to $E\{|v(n)|^2\} = \sigma_v^2$. The total variance of the DFT calculated for CS signal with *M* samples is $\text{var}\{X(k)\} = M\sigma_v^2$. After components detection, the inversion of the signal is done by:

$$x_R(n)=\frac{1}{N}\sum_{k=1}^{K} e^{j2\pi kn/N}[X_x(k)+X_v(k)], \tag{21}$$

where $X_x(k)$ is deterministic component belonging to the signal, while $X_v(k)$ is a part of DFT belonging to the noise in detected signal components. The variance in the reconstructed signal caused by the noise component is:

$$\text{var}\{x_R(n)\}=\frac{1}{N^2}K\,\text{var}\{\frac{N}{M}X_{noise}(k_m)\}=\frac{K}{N^2}\frac{N^2}{M^2}M\sigma_v^2=\frac{K}{M}\sigma_v^2 . \tag{22}$$

Note that the CS signal has the DFT amplitudes MA_i, which after the full signal reconstruction becomes NA_i. It means that during the reconstruction, the DFT is scaled by the factor N/M, including the DFT noise component, as given in (22). Finally, the signal to noise ratio of the reconstructed signal is obtained as:

$$SNR_R=10\log\frac{A_1^2+A_2^2+...+A_K^2}{(K/M)\sigma_v^2}=SNR_i-10\log\frac{K}{M}, \tag{23}$$

where SNR_R is the signal to noise ratio of the reconstructed signal, while the SNR_i is the signal to noise ratio of the original full signal. Therefore, the expression (23) shows that the input SNR_R is improved for $-10\log(K/M)$ dB compared to SNR_i, since $K<<M$, meaning a significant improvement of the accuracy toward the non-noisy signal version. The experimental evaluation of this result is provided in Example 2.

4. EXPERIMENTAL EVALUATION

Example 1: Let us observe a multicomponent signal:

$$s(n)=A_1\exp(j2\pi nf_1/N)+A_2\exp(j2\pi nf_2/N)+A_3\exp(j2\pi nf_3/N)+A_4\exp(j2\pi nf_4/N)+\nu(n)$$

where A_1=4.5, A_2=4, A_3=2, A_4=1.75, while the frequencies of the components are: f_1=32, f_2=126, f_3=228, f_4=356. Also, the complex Gaussian noise $\nu(n)$ with $\sigma^2_N\approx 8$ is assumed, producing SNR=7.5dB on the available set of measurements. The total number of samples is N=512 and the number of available samples $Mopt$=192 is used. The probability of error is set as: $P(T)$=0.99. The single-iteration solution provided by ***Algorithm 1*** is applied. The illustration of thresholding procedure and the reconstructed DFT are shown in Fig. 3.

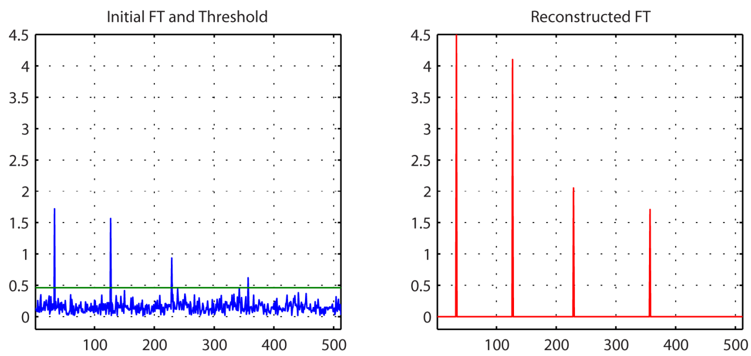

Fig. 3. Reconstruction results achieved using Algorithm 1

The same procedure can be applied for larger number of components K as illustrated in Fig. 4 (12 components are considered), as long as $K<<M$ holds.

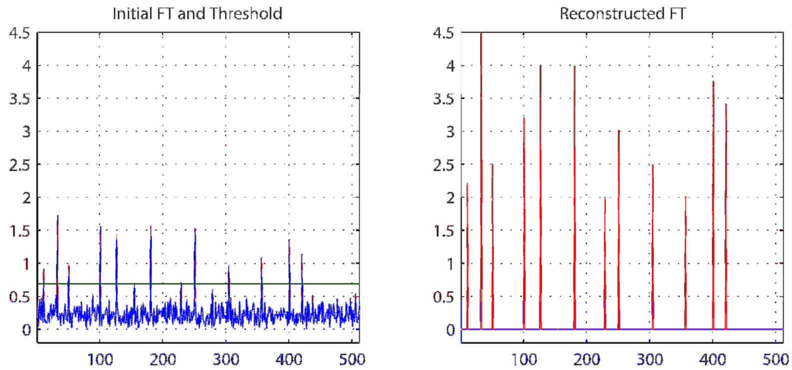

Fig. 4. Reconstruction results achieved using the procedure defined by Algorithm 1: A=[4.5, 4, 2, 2, 2.5, 4, 3, 2.5, 2.2, 3.2, 3.75, 3.4], f=[32, 126, 228, 356, 50, 180, 250, 300, 10, 100, 400, 420]

Note that, for the sake of simplicity, we have considered examples with frequencies on-grid. Generally, the signal can be considered as sparse only when the frequencies are on-grid. However, the same concept (with slight modifications) can be applied for off-grid frequency components. The signal should be firstly windowed, for example by using the Hamming window, which will improve the Fourier transform convergence and decrease the number of non-zero components in the spectrum when the frequencies are not on grid. After the thresholding and appropriate reconstruction, the side components can be used to reconstruct the true position and the DFT coefficient value, by using the spectral displacement bin for employed window function, as it was done in [3], [25].

The proposed method provides two important advantages over the other greedy methods such as the OMP. Namely, 1) unlike the other methods, the proposed one is, in essence, a single iteration method which can recover all components at once with minimal computational complexity; 2) Even if the components are masked by strong noise, the proposed method requires just a couple of iterations, which is again lower than in the OMP; 3) The proposed method is blind in terms of the number of components, i.e., it does not require any assumption about the number of components.

Example 2: The experimental evaluation on the achieved SNR_R of the reconstructed signal is done based on the statistical results within 1000 repetitions of the reconstruction procedure (for a predefined SNR_i≈6dB and SNR_i≈3dB). The cases with K=3 (A_1=4.5, A_2=4, A_3=2) and K=4 (A_1=4.5, A_2=4, A_3=2, A_4=1.75) are observed. The parameters used in the simulation and the statistical results are given in the Table 2. The statistical results confirm that for each pair (K,M), the SNR_R is reduced for exactly 10log(K/M) compared to SNR_i.

TABLE 2. THE EXPERIMENTAL RESULTS FOR THE SNR_R

	M=256	M=384	M=256	M=384
K=3	$SNRi$=6dB	$SNRi$=6dB	$SNRi$=3 dB	$SNRi$=3dB
	SNR_R=25.3dB	SNR_R=27.2dB	SNR_R=22.3dB	SNR_R=24.1dB
K=4	$SNRi$=6.3dB	$SNRi$=6.3dB	$SNRi$=3.35dB	$SNRi$=3.35dB
	SNR_R=24.4dB	SNR_R=26.3dB	SNR_R=21.2dB	SNR_R=23.2dB

Example3: The same noisy signal with significantly reduced set of observations M=100 (from N=512) is considered. In this case, some of the signal components with lower amplitudes are highly embedded in noise. Hence, the iterative procedure defined by the ***Algorithm 2*** is applied. All four components are selected within two iterations (Fig. 5). The final result of CS reconstruction is equal to original DFT.

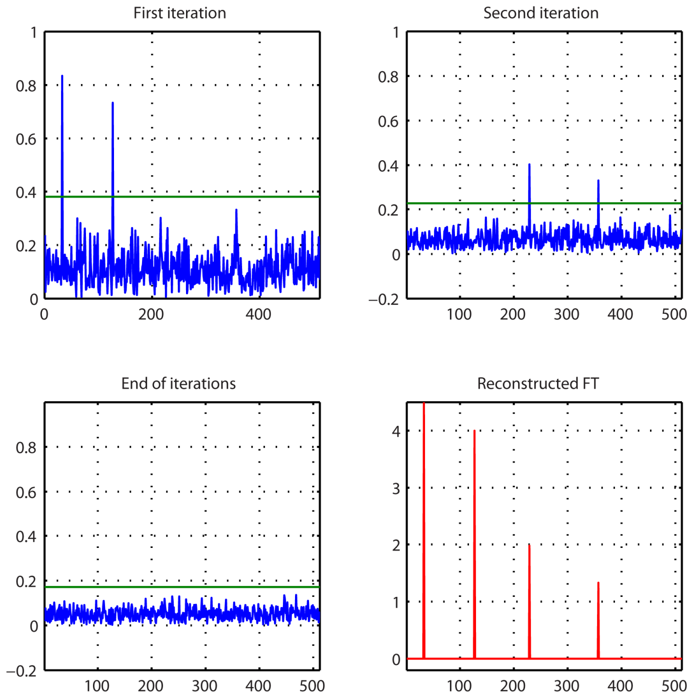

Fig. 5. Iterations and thresholding using the procedure defined by Algorithm 2

5. CONCLUSION

The proposed analysis reveals two important issues that appear in the applications dealing with CS signals. The first issue is related to the number of measurements that are required for successful reconstruction of sparse signals. It is shown that this number directly affects the performance of signal reconstruction through the phenomenon of induced noise. In the presence of additional external noise, the required number of measurements increases and can be determined by using the proposed expression. The second issue resolves the possibility of reliable signal components detection and reconstruction in the presence of noise. Hence, for a calculated number of samples, we can set the threshold to detect signal components, and provide simple signal reconstruction algorithm.

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