# System Architecture for Space-Frequency Image Analysis

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*Abstract*— **A system architecture for space-frequency image analysis is presented. The system is based on the two dimensional S-method with variable window width in the frequency domain.**

## I. INTRODUCTION

The short time Fourier transform and the Wigner distribution can be used for space-frequency analysis of two dimensional nonstationary signals [1], [2], [3]. However, the short time Fourier transform has a low concentration around the local frequency, while the Wigner distribution has a good concentration but its serious drawback lies in the presence of cross terms in the case of multicomponent signals. Using 2D S-method, the shortcomings of both distributions can be overcome [4]. The method is efficient for numerical calculation and particularly suitable for hardware realization. This property is very important, because analysis of the image through 2D space-frequency distributions needs a significant computer calculation time. This Letter presents system architecture for space-frequency signal analysis.

## II. TWO DIMENSIONAL S METHOD

The 2D S method is defined as [4]

$$SM(n_1,n_2,k_1,k_2) = \sum_{i_1=-L_1}^{L_1}\sum_{i_2=-L_1}^{L_1} P(i_1,i_2)$$
$$\times STFT(n_1,n_2,k_1+i_1,k_2+i_2)$$
$$\times STFT^*(n_1,n_2,k_1-i_1,k_2-i_2) \qquad (1)$$

where $STFT$ denotes the short time Fourier transform defined as

$$STFT(n_1,n_2,k_1,k_2) =$$



$$\sum_{i_1=-(N/2)+1}^{N/2}\sum_{i_2=-(N/2)+1}^{N/2} f(n_1+i_1,n_2+i_2)$$
$$\times w^*(i_1,i_2)e^{-j(2\pi/N)(i_1k_1+i_2k_2)}$$

Note that, by taking $P(i_1,i_2)=1$ and $2L_1+1=2L_2+1=N$ the Wigner distribution is obtained. Also, if we take $P(i_1,i_2)=\delta(i_1,i_2)$ the spectrogram follows. Window width is taken as $(2L_1+1)\times(2L_2+1)$.

Suitable choice of the window $P(i_1,i_2)$ will lead to a significant reduction or completed elimination of, while the concentration of distribution remains the same as that in Wigner distribution. In the S method, calculation of oversampling, needed in Wigner distribution calculation, is not necessary. More details about the S method in 1D and 2D cases may be found in [4], [5]. Relation (1) for $P(i_1,i_2)=1$ become:

$$SM(n_1,n_2,k_1,k_2) = |STFT(n_1,n_2,k_1,k_2)|^2+$$
$$2\sum_{i_1=0}^{L}\sum_{i_2=1}^{L}\{STFT(n_1,n_2,k_1+i_1,k_2+i_2)$$
$$\times STFT(n_1,n_2,k_1+i_1,k_2+i_2)\}+$$
$$2\sum_{i_1=1}^{L}\sum_{i_2=-L}^{0}\{STFT(n_1,n_2,k_1+i_1,k_2+i_2)$$
$$\times STFT^*(n_1,n_2,k_1-i_1,k_2-i_2)\} \qquad (2)$$

where $L_1=L_2=L$ and $L<(N-1)/2$ is assumed.

## III. ARCHITECTURE FOR THE SYSTEM REALIZATION

Considering the relation for the S method we can conclude that window $P(i_1,i_2)$ must be chosen to provide summation over $STFT(n_1,n_2,k_1+i_1,k_2+i_2)$, and $STFT^*(n_1,n_2,k_1-i_1,k_2-i_2)$, while both of

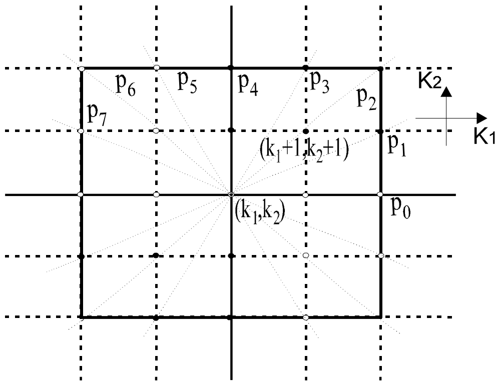


Fig. 1. Summation lines for $L = 2$

them are different from zero. Bearing in mind that we work in discrete domain, one possible scheme to performing the summation is through the lines shown in Fig. 1.

Thus, through the lines $P$ every point of the $STFT$ will be included, and a summation according to (2) will be performed. In this way we introduce signal dependent 2D window. Note that the dimensions of this window depend on the signal's spectral components ($STFT$).

Considering the lines $P$ given in Figure 1, we can conclude that every line may be defined in general form through the points $(a, b)$ so that $(a, b) \in S \cup (0, 1) \cup (1, 0)$ were $S$ represents a subset of prime numbers, where $b > 0$. According to the previous consideration, S method can be written in the shortened form as

$$SM(n_1, n_2, k_1, k_2) = |STFT(n_1, n_2, k_1, k_2)|^2 + 2 \sum_{(a,b)} \sum_{i=1}^{L_{(a,b)}} P_{n_1,n_2}(k_1, k_2, ai, bi) \times \mathrm{Re}\{STFT(n_1, n_2, k_1 + ai, k_2 + bi) \times STFT^*(n_1, n_2, k_1 - ai, k_2 - bi)\}$$

Thus, we will perform the summation along any lines $P$ until both $STFT(n_1, n_2, k_1 + ai, k_2 + bi)$ and $STFT^*(n_1, n_2, k_1 - ai, k_2 - bi)$ are non-zero ($i = L_{(a,b)}$ is the border point for $(a, b)$ $L_{(a,b)} \leq L_{max}/(a^2 + b^2)$). Obviously, the total summation will be performed inside the 2D window with variable size. Note that cross terms are eliminated by the signal dependent window if no point of another $STFT$ component is not included in summation. This condition is fulfilled for all cases when $STFT$s are not overlapped. Also, if we introduce a reference level $R_{n_1,n_2}$, so that $STFT(n_1, n_2, k_1, k_2) = 0$ for $STFT(n_1, n_2, k_1, k_2) < R_{n_1,n_2}$ the influence of the noise will be significantly reduced. Index in $R$ is added to show that this value depends on $n_1$, $n_2$. The reference value may be defined as: $R_{n_1,n_2} = \max_{k_1,k_2} |STFT(n_1, n_2, k_1, k_2)|/Q$ were is $1 \leq Q < \infty$. In our examples $Q = 2$ is used.

The most important point in the previous consideration is its suitability for hardware realization. By introducing comparators circuit with a binary signal on the output, it is possible to use logic circuits to control of summation. The signals on the comparator's output is defined as

$$X_{k_1k_2} = \begin{cases} 1, & |STFT(n_1, n_2, k_1, k_2)| > R_{n_1n_2} \\ 0, & |STFT(n_1, n_2, k_1, k_2)| < R_{n_1n_2} \end{cases}$$

$k_1 = 1, 2, \ldots, L$, $k_2 = 1, 2, \ldots, L$. The control signals for including or excluding some line in summation are

$$C_{al,bl} = \prod_{i=1}^{l \leq L_{(a,b)}} X_{k_1+ai,k_2+bi} X_{k_1-ai,k_2-bi}$$

It is very easy to realize this relation using the logic circuits. We can use control sig-

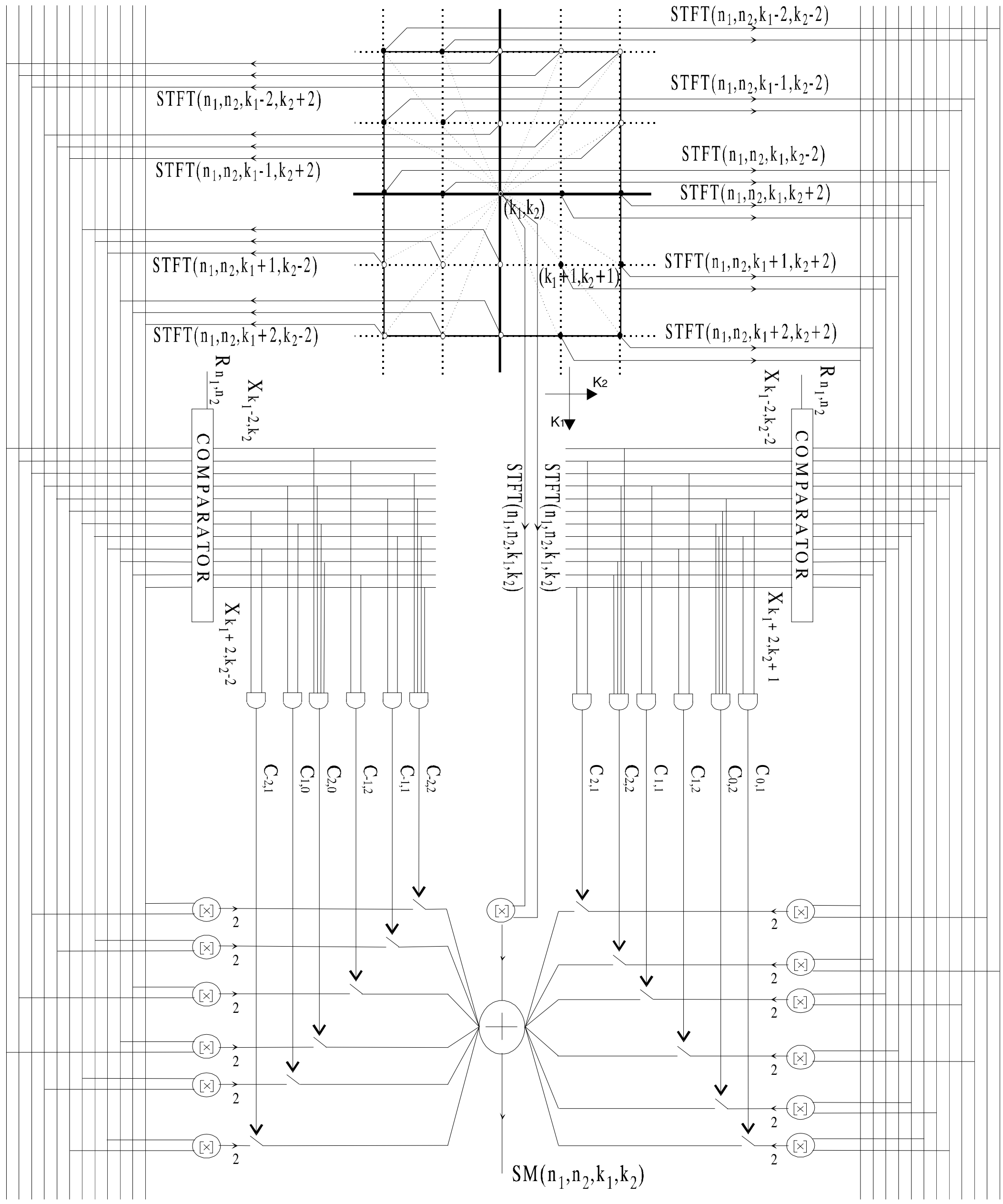


Fig. 2. Architecture for twodimensional adaptive S-Method realization

nals for switch's circuits; if the switch is closed (logic 1 on it) then the product of the pair of $STFT$s will be transferred along the line of this switch to the input of the adder. In this way, all lines with closed switcher will be included in summation. Note that one line is not connected through switch. Product of this line is the spectrogram and it will be transferred to the output in any case. The architecture of the system is presented in Fig. 2.

It is very suitable for ASIC implementation [6]. Previous architectures were designed only for the point $(k_1, k_2)$, the complete architecture will be obtained by using of the presented

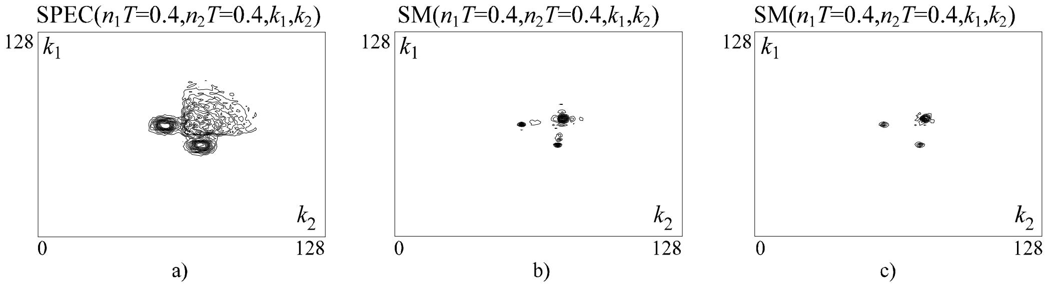


Fig. 3. a) Spectrogram; b) S-Method with constant window width c) S-Method with adaptive window width

system for every point.

## IV. EXAMPLE

To illustrate the procedure, we take the signal

$$f(x, y) = \exp(j42\pi(x^3 + y^3))$$
$$+\cos(20\pi x^2 - 16\pi y^2) + \nu(x, y)$$

where $\nu(x, y)$ is a white additive Gaussian noise with the variance $\sigma^2_{\nu\nu} = 0.6$. The signal is sampled with the period $(\Delta x, \Delta y) = (1/128, 1/128)$ and observed at point $(x, y) = (0.4, 0.4)$.

Fig. 3 shows the SPEC, S-method with constant window width for $L = 4$, and the S-method with signal-adaptive window width for $Q = 2$. The Figure shows that the component concentration in the adaptive S-method is better than in the S-method with constant window width, while the number of symmetric components of the $STFT$ used for calculations in the S-method is smaller for the adaptive S-method than for the S-method with a constant window width. In our example 5483 components are added to the spectrogram for adaptive S-method, and 590480 components for the S-method with a constant window width.

## V. CONCLUSION

A system architecture for space-frequency analysis is presented. This system is suitable for hardware realization and ASIC implementation. This is important because space-frequency analysis of an image needs a large amount of computer time, so that real time implementation will make space-frequency distribution attractive for this purpose.

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